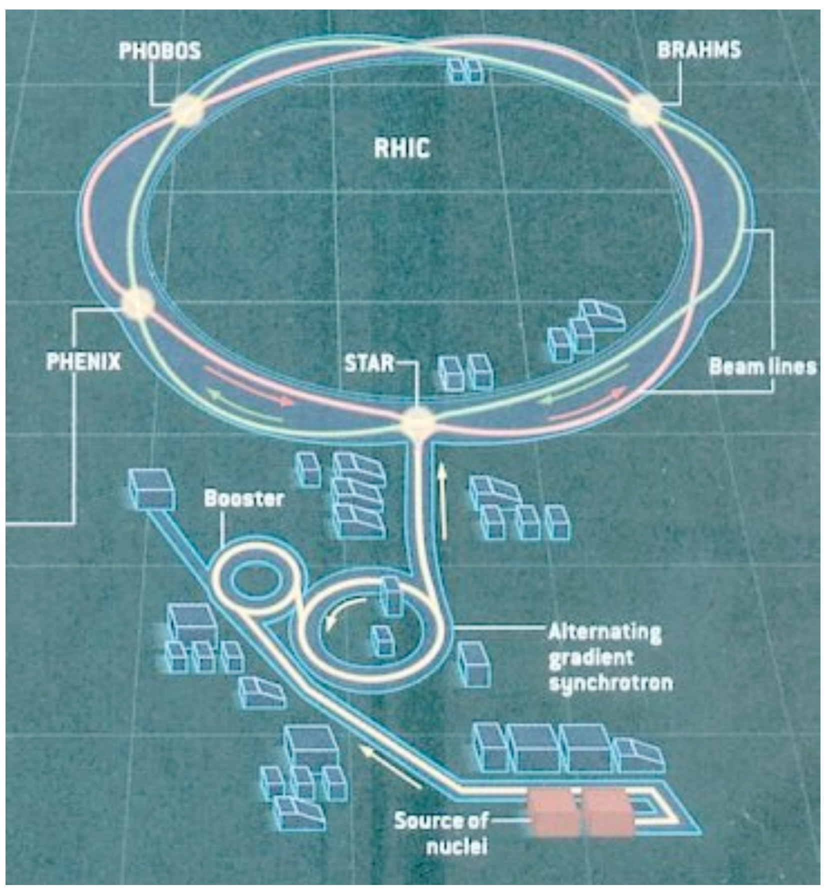
Elimination of QCD Scale Ambiguities

The Principle of Maximum Conformality (PMC), and Novel QCD Effects

11th International Workshop on High P_T in the RHIC and LHC Era

April 12, 2016



Stan Brodsky

SLAC
NATIONAL ACCELERATOR LABORATORY

LELAND STANFORD JUNIOR UNIVERSITY
DIE LUFT DER FREIHEIT WEHT
1891

with Leonardo Di Giustino , Xing-Gang Wu, and Matin Mojaza

Goals

- **Test QCD to maximum precision at colliders**
- **Maximize sensitivity to new physics**
- **Obtain high precision determination of fundamental parameters**
- **Determine renormalization scales without ambiguity**
- **Eliminate scheme dependence**

Predictions for physical observables cannot depend on theoretical conventions such as the renormalization scheme or initial scale choice

Lessons from QED

- **No Renormalization Scale Ambiguity**
- **Dressed Photon Propagator sums all β terms**
- **New Scale at Every Order, Every Skeleton Graph**
- **Predictions are scheme independent**
- **QCD becomes Abelian QED in Zero Color Limit** $N_C \to 0$
- **Grand Unification: Use same methods for all couplings**

Can use $\overline{MS}$ scheme in QED; answers are scheme independent
Analytic extension: coupling is complex for timelike argument

Electron-Electron Scattering in QED

$$\mathcal{M}_{ee\to ee}(++;++) = \frac{8\pi s}{t}\,\alpha(t) + \frac{8\pi s}{u}\,\alpha(u)$$

$$\alpha(t) = \frac{\alpha(0)}{1-\Pi(t)}$$

Gell-Mann--Low Effective Charge

- **Dressed Photon Propagator sums all β (vacuum polarization) contributions, proper and improper**

$$\alpha(t) = \frac{\alpha(t_0)}{1-\Pi(t,t_0)} \qquad \Pi(t,t_0) = \frac{\Pi(t)-\Pi(t_o)}{1-\Pi(t_0)}$$

- **Initial Scale Choice t_o is Arbitrary!**

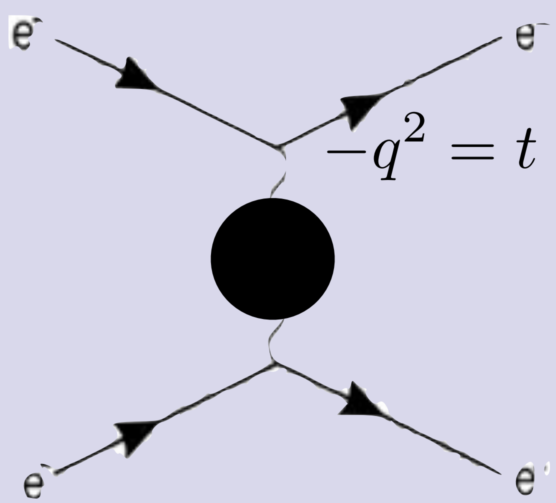
- **Any renormalization scheme can be used** $\alpha(t) \to \alpha_{\overline{MS}}(e^{-\frac{5}{3}}t)$

Lessons from QED

In the (physical) Gell Mann-Low scheme, the momentum scale of the running coupling is the virtuality of the exchanged photon; independent of initial scale.

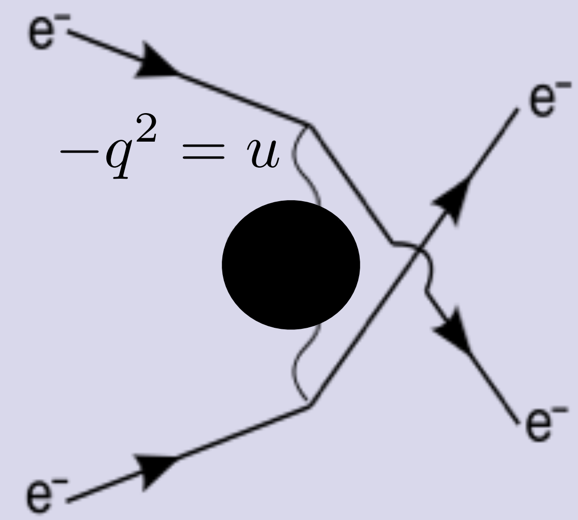
$$\alpha(t) = \frac{\alpha(t_0)}{1 - \Pi(t, t_0)} \qquad \Pi(t, t_0) = \frac{\Pi(t) - \Pi(t_0)}{1 - \Pi(t_0)}$$

Example: ee-scattering



$$\mathcal{M}_{ee\to ee} = \frac{8\pi s}{t}\alpha(t) + \frac{8\pi s}{u}\alpha(u)$$

Two separate scales;
one for each skeleton graph.



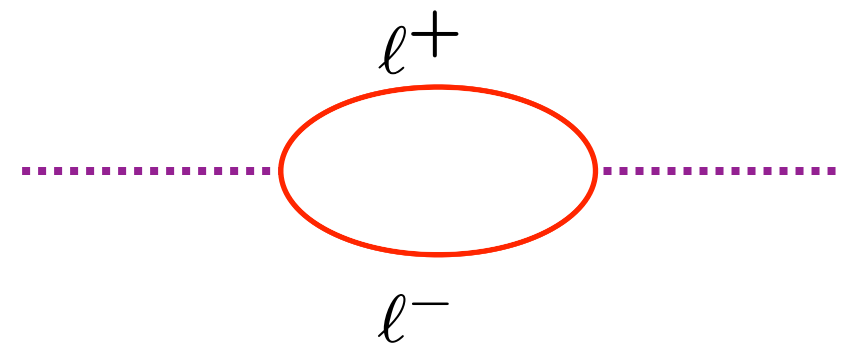
For any other scale choice an infinite set of diagrams must be taken into account to obtain the correct result!

In any other scheme, the correct scale displacement must be used

$$\log \frac{\mu^2_{\overline{MS}}}{m_\ell^2} = 6\int_0^1 dx\, x(1-x) \log \frac{m_\ell^2 + Q^2 x(1-x)}{m_\ell^2}, \quad \xrightarrow{Q^2 \gg m_\ell^2} \log \frac{Q^2}{m_\ell^2} - \frac{5}{3}$$

$$\boxed{\alpha_{\overline{MS}}(e^{-5/3} q^2) = \alpha_{GM-L}(q^2).}$$

QED One-Loop Vacuum Polarization



$t = -Q^2 < 0$

(t spacelike)

$$\Pi(Q^2) = \frac{\alpha(0)}{3\pi}\left[\frac{5}{3} - \frac{4m^2}{Q^2} - \left(1 - \frac{2m^2}{Q^2}\right)\sqrt{1 + \frac{4m^2}{Q^2}}\,\log\frac{1+\sqrt{1+\frac{4m^2}{Q^2}}}{\left|1-\sqrt{1+\frac{4m^2}{Q^2}}\right|}\right]$$

Analytically continue to timelike t: Complex

$$\Pi(Q^2) = \frac{\alpha(0)}{15\pi}\frac{Q^2}{m^2} \qquad Q^2 << 4M^2$$

Serber-Uehling

$$\Pi(Q^2) = \frac{\alpha(0)}{3\pi}\log\frac{Q^2}{m^2} \qquad Q^2 >> 4M^2$$

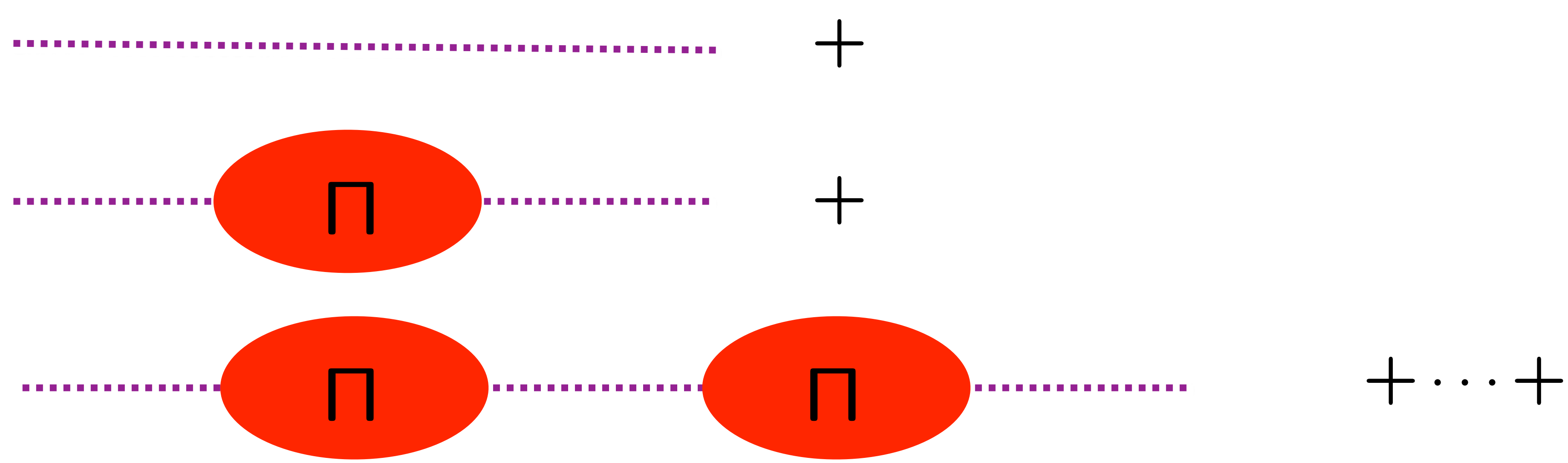
Potential Landau Pole

$$\beta_{QED}(t) = \frac{1}{4\pi}\frac{d\,\alpha(t)}{d\log t} = \frac{4}{3}\left(\frac{\alpha}{4\pi}\right)^2 n_\ell \geq 0$$

QED Running Coupling

$$\alpha(t) = \frac{\alpha(0)}{1 - \Pi(t)}$$

All-orders lepton-loop corrections to dressed photon propagator



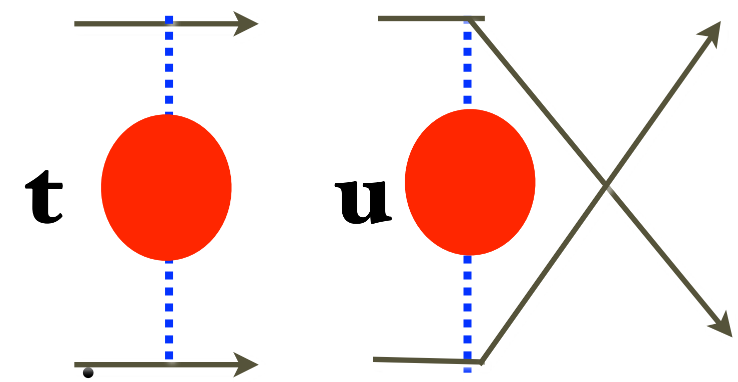
$$\alpha(t) = \frac{\alpha(t_0)}{1 - \Pi(t, t_0)} \qquad \Pi(t, t_0) = \frac{\Pi(t) - \Pi(t_0)}{1 - \Pi(t_0)}$$

Initial **scale t_o is arbitrary -- Variation gives RGE Equations**

Physical renormalization scale t not arbitrary!

Electron-Electron Scattering in QED

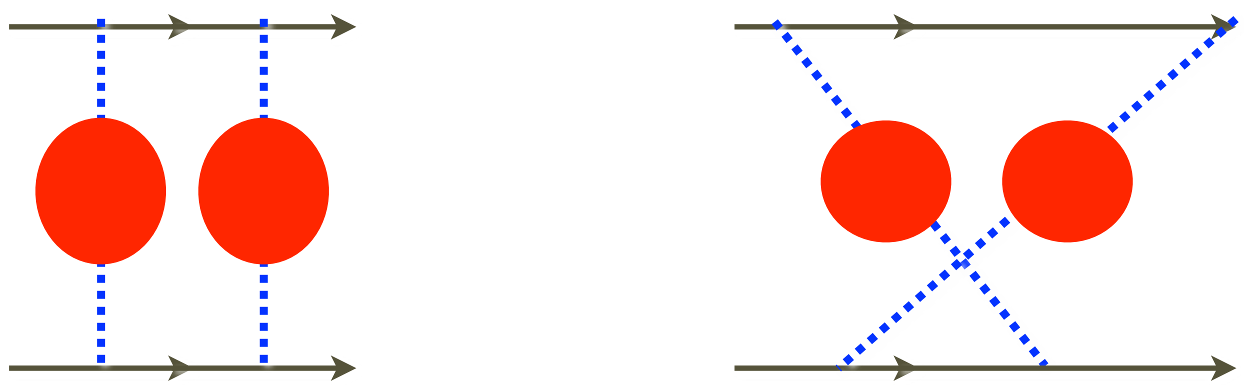
$$\mathcal{M}_{ee\to ee}(++;++) = \frac{8\pi s}{t}\,\alpha(t) + \frac{8\pi s}{u}\,\alpha(u)$$



- **No renormalization scale ambiguity!**
- **Gauge Invariant. Dressed photon propagator**
- **Sums all vacuum polarization, non-zero beta terms into running coupling. This is the purpose of the running coupling!**
  - **Two separate physical scales: t, u = photon virtuality**
- **If one chooses a different initial scale, one must sum an infinite number of graphs -- but always recover same result!**
- **Number of active leptons correctly set**
- **Analytic: reproduces correct behavior at lepton mass thresholds**
- **No renormalization scale ambiguity!**

Electron-Electron Scattering in QED

New renormalization scale at each order of pQED



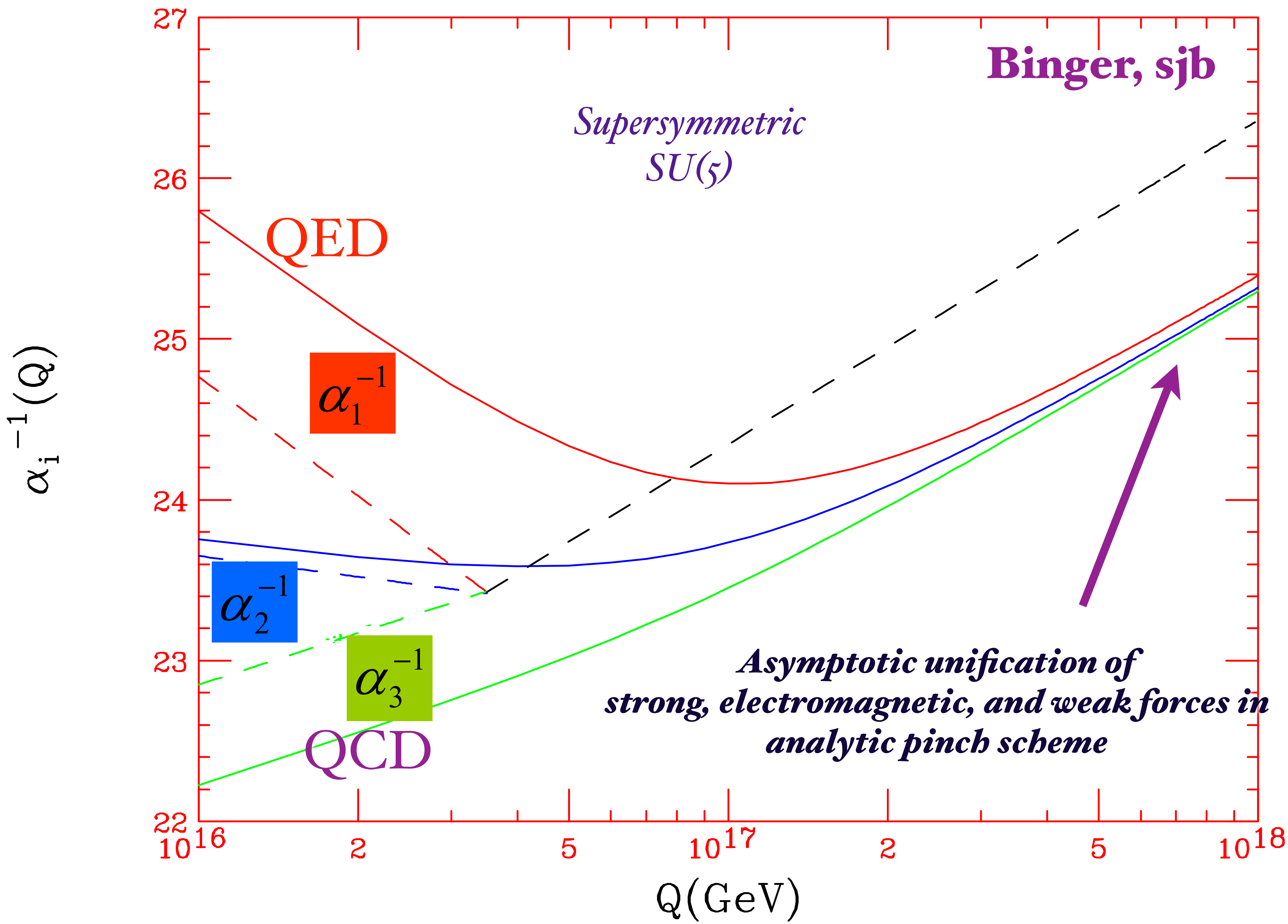
Each "skeleton" graph has its own renormalization scale

Renormalization scheme independent at each order

Independent of initial scale μ_0

Abelian theory is the analytic limit QCD at $N_c = 0$

GUT: Must use the same scale - setting procedure for QED, QCD



$$C_F = \frac{N_C^2 - 1}{2N_C}$$

Huet, sjb

$$\boxed{\lim N_C \to 0 \text{ at fixed } \alpha = C_F \alpha_s, n_\ell = n_F / C_F}$$

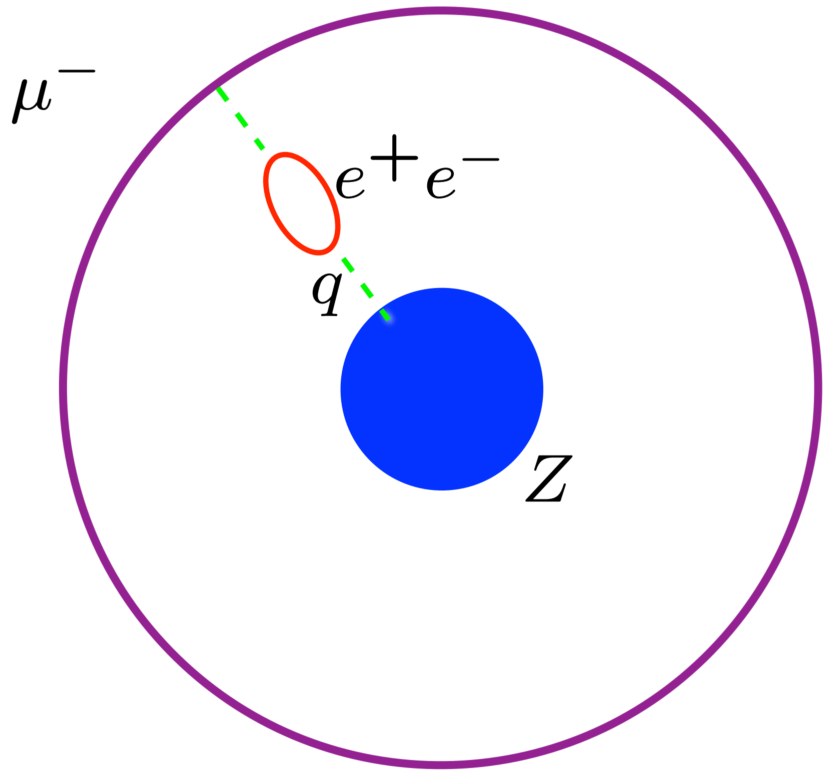
$$\boxed{\text{QCD} \to \text{Abelian Gauge Theory}}$$

Analytic Feature of SU(Nc) Gauge Theory

Scale-Setting procedure for QCD must be applicable to QED

- No renormalization scale ambiguity in QED
- No guessing of renormalization scale or range!
- Physical predictions cannot depend on renormalization scheme
- Gell Mann-Low QED Coupling defined from physical observable
- Running Coupling sums all Vacuum Polarization Contributions, all β terms
- Recover conformal series
- Renormalization Scale in QED scheme: Identical to Photon Virtuality
- Analytic: Reproduces lepton-pair thresholds -- number of active leptons set
- Examples: muonic atoms, g-2, Lamb Shift
- Time-like and Space-like QED Coupling related by analyticity
- Dressed Skeleton Expansion

Example in QED: Muonic Atoms



$$V(q^2) = -\frac{Z\alpha_{QED}(q^2)}{q^2}$$

$$\mu_R^2 \equiv q^2$$

$$\alpha_{QED}(q^2) = \frac{\alpha_{QED}(0)}{1-\Pi(q^2)}$$

Scale is unique: Tested to ppm

Gyulassy: Higher Order VP verified to 0.1% precision in μ Pb

Principle of Maximum Conformality (PMC)

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Systematic All-Orders Method to Eliminate Renormalization-Scale and Scheme Ambiguities in Perturbative QCD

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We introduce a generalization of the conventional renormalization schemes used in dimensional regularization, which illuminates the renormalization scheme and scale ambiguities of perturbative QCD predictions, exposes the general pattern of nonconformal $\{\beta_i\}$ terms, and reveals a special degeneracy of the terms in the perturbative coefficients. It allows us to systematically determine the argument of the running coupling order by order in perturbative QCD in a form which can be readily automatized. The new method satisfies all of the principles of the renormalization group and eliminates an unnecessary source of systematic error.

Features of BLM/PMC

- ***Predictions are scheme-independent at every order***
- ***Matches conformal series***
- ***No n! Renormalon growth of pQCD series***
- ***New scale appears at each order; n_F determined at each order - matches virtuality of quark loops***
- ***Multiple Physical Scales Incorporated (Hoang, Kuhn, Tuebner, sjb)***
- ***Rigorous: Satisfies all Renormalization Group Principles***
- ***Realistic Estimate of Higher-Order Terms***
- ***Reduces to standard QED scale*** $N_C \to 0$
- ***GUT: Must use the same scale setting procedure for QED, QCD***
- ***Eliminates unnecessary theory error***
- ***Maximal sensitivity to new physics***
- ***Commensurate Scale Relations between observables: Generalized Crewther Relation (Kataev, Lu, Rathsman, sjb)***
- ***Reduces to BLM at NLO: Example: BFKL intercept*** **(Fadin, Kim, Lipatov, Pivovarov, sjb)**

BLM/PMC: Set Scales

such to absorb all 'renormalon-terms', i.e. non-conformal terms

$$\begin{aligned}\rho(Q^2) = r_{0,0} &+ r_{1,0}a(Q) + (\beta_0 a(Q)^2 + \beta_1 a(Q)^3 + \beta_2 a(Q)^4 + \underline{\cdots)r_{2,1}} \\ &+ (\beta_0^2 a(Q)^3 + \frac{5}{2}\beta_1\beta_0 a(Q)^4 + \underline{\cdots)r_{3,2}} + (\beta_0^3 + \cdots)r_{4,3} \\ &+ r_{2,0}a(Q)^2 + 2a(Q)(\beta_0 a(Q)^2 + \beta_1 a(Q)^3 + \underline{\cdots)r_{3,1}} \\ &+ \cdots\end{aligned}$$

$$r_{1,0}a(Q_1) = r_{1,0}a(Q) - \beta(a)r_{2,1} + \frac{1}{2}\beta(a)\frac{\partial\beta}{\partial a}r_{3,2} + \cdots + \frac{(-1)^n}{n!}\frac{d^{n-1}\beta}{(d\ln\mu^2)^{n-1}}r_{n+1,n}$$

$$r_{2,0}a(Q_2)^2 = r_{2,0}a(Q)^2 - 2a(Q)\beta(a)r_{3,1} + \cdots$$

How do we identify the β terms?

BLM: Use n_f dependence of β_0 and β_1

Principle of Maximum Conformality (PMC)

- Subtract extra constant δ in dimensional regularization. Defines new scheme R_δ

$$\log 4\pi - \gamma_E - \delta \qquad \overline{MS} : \delta = 0$$

(δ: Arbitrary constant!)

- $\boxed{\text{Coefficients of δ identify β terms !}}$

- Shift β terms to argument of running coupling $\alpha_s(Q_n^2)$ at each order n (analogous to all-orders vacuum polarization summation in QED)

- Resulting PQCD series matches β= 0 conformal series

- scheme-independent predictions at each computed order

- almost independent of initial scale μ_0

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δ-$\mathcal{R}$enormalization Scheme ($\mathcal{R}_\delta$ scheme)

In dim. reg. $1/\epsilon$ poles come in powers of [Bollini & Gambiagi, 't Hooft & Veltman, '72]

$$\ln \frac{\mu^2}{\Lambda^2} + \frac{1}{\epsilon} + c$$

In the modified minimal subtraction scheme (MS-bar) one subtracts together with the pole a constant [Bardeen, Buras, Duke, Muta (1978) on DIS results]:

$$\ln(4\pi) - \gamma_E$$

This corresponds to a shift in the scale:

$$\mu^2_{\overline{\mathrm{MS}}} = \mu^2 \exp(\ln 4\pi - \gamma_E)$$

A finite subtraction from infinity is arbitrary. *Let's make use of this!*

Subtract an arbitrary constant and keep it in your calculation: $\mathcal{R}_\delta$-scheme

$$\ln(4\pi) - \gamma_E - \delta,$$

$$\mu^2_\delta = \mu^2_{\overline{\mathrm{MS}}} \exp(-\delta) = \mu^2 \exp(\ln 4\pi - \gamma_E - \delta)$$

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Teach a robot to compute the PMC scales

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Generalize $\overline{MS}$ Scheme by subtracting $\log 4\pi - \gamma_E - \delta$

Call this the $\mathcal{R}_\delta$ renormalization scheme

$$\mathcal{R}_0 = \overline{\text{MS}} \; ,$$
$$\mathcal{R}_{\ln 4\pi - \gamma_E} = \text{MS} \; .$$

All $\mathcal{R}_\delta$ renormalization schemes have same β-function

$$\mu_{\delta_2} = \mu_{\delta_1} e^{\frac{\delta_1 - \delta_2}{2}} \; .$$

In particular:

$$\mu_{\overline{\text{MS}}} = \mu_{\text{MS}} \; e^{(\ln 4\pi - \gamma_E)/2},$$
$$\mu_\delta = \mu_{\overline{\text{MS}}} \; e^{-\delta/2} \; .$$

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Exposing the Renormalization Scheme Dependence

Observable in the $\mathcal{R}_\delta$ -scheme:

$$\rho_\delta(Q^2) = r_0 + r_1 a(\mu) + [r_2 + \beta_0 r_1 \delta] a(\mu)^2 + [r_3 + \beta_1 r_1 \delta + 2\beta_0 r_2 \delta + \beta_0^2 r_1 \delta^2] a(\mu)^3 + \cdots$$

$$\mathcal{R}_0 = \overline{\mathrm{MS}} \ , \quad \mathcal{R}_{\ln 4\pi - \gamma_E} = \mathrm{MS} \qquad \mu^2 = \mu^2_{\overline{\mathrm{MS}}} \ \exp(\ln 4\pi - \gamma_E) \ , \quad \mu^2_{\delta_2} = \mu^2_{\delta_1} \ \exp(\delta_2 - \delta_1)$$

Note the divergent 'renormalon series' $n! \beta^n \alpha_s^n$

Renormalization Scheme Equation

$$\frac{d\rho}{d\delta} = -\beta(a) \frac{d\rho}{da} \stackrel{!}{=} 0 \quad \longrightarrow \mathrm{PMC}$$

$$\rho_\delta(Q^2) = r_0 + r_1 a_1(\mu_1) + (r_2 + \beta_0 r_1 \delta_1) a_2(\mu_2)^2 + [r_3 + \beta_1 r_1 \delta_1 + 2\beta_0 r_2 \delta_2 + \beta_0^2 r_1 \delta_1^2] a_3(\mu_3)^3$$

The $\delta_k^p a^n$-term indicates the term associated to a diagram with $1/\epsilon^{n-k}$ divergence for any p. Grouping the different δ_k-terms, one recovers in the $N_c \to 0$ Abelian limit the dressed skeleton expansion.

Coefficients of δ identify β_i and their pattern

Special Degeneracy in PQCD

There is nothing special about a particular value for δ, thus for any δ

$$\rho(Q^2) = r_{0,0} + r_{1,0}a(Q) + [r_{2,0} + \beta_0 \underline{r_{2,1}}]a(Q)^2 + [r_{3,0} + \beta_1 \underline{r_{2,1}} + 2\beta_0 \underline{r_{3,1}} + \beta_0^2 \underline{r_{3,2}}]a(Q)^3$$
$$+ [r_{4,0} + \beta_2 \underline{r_{2,1}} + 2\beta_1 \underline{r_{3,1}} + \frac{5}{2}\beta_1\beta_0 \underline{r_{3,2}} + 3\beta_0 r_{4,1} + 3\beta_0^2 r_{4,2} + \beta_0^3 r_{4,3}]a(Q)^4$$

General pattern of pQCD

According to the principal of maximum conformality we must set the scales such to absorb all 'renormalon-terms', i.e. non-conformal terms

$$\rho(Q^2) = r_{0,0} + r_{1,0}a(Q) + (\beta_0 a(Q)^2 + \beta_1 a(Q)^3 + \beta_2 a(Q)^4 + \cdots)r_{2,1}$$
$$+ (\beta_0^2 a(Q)^3 + \frac{5}{2}\beta_1\beta_0 a(Q)^4 + \cdots)r_{3,2} + (\beta_0^3 + \cdots)r_{4,3}$$
$$+ r_{2,0}a(Q)^2 + 2a(Q)(\beta_0 a(Q)^2 + \beta_1 a(Q)^3 + \cdots)r_{3,1}$$
$$+ \cdots$$

$$r_{1,0}a(Q_1) = r_{1,0}a(Q) - \beta(a)r_{2,1} + \frac{1}{2}\beta(a)\frac{\partial\beta}{\partial a}r_{3,2} + \cdots + \frac{(-1)^n}{n!}\frac{d^{n-1}\beta}{(d\ln\mu^2)^{n-1}}r_{n+1,n}$$

$$r_{2,0}a(Q_2)^2 = r_{2,0}a(Q)^2 - 2a(Q)\beta(a)r_{3,1} + \cdots$$

Since ρ is a physical observable, it must be independent of the arbitrary renormalization scheme and scale. That is,

initial

$$\boxed{\frac{\partial \rho_\delta}{\partial \mu_\delta} = 0\ , \qquad \frac{\partial \rho_\delta}{\partial \delta} = 0\ ,} \tag{16}$$

Generalization: use δ_n at n-loops.

$$\begin{aligned}\rho_\delta(Q^2) =& r_0 + r_1 a_1(Q) + (r_2 - \beta_0 r_1 \delta_1) a_2(Q)^2 \\ &+ [r_3 - \beta_1 r_1 \delta_1 - 2\beta_0 r_2 \delta_2 + \beta_0^2 r_1 \delta_1^2] a_3(Q)^3 \\ &+ [r_4 - \beta_2 r_1 \delta_1 - 2\beta_1 r_2 \delta_2 - 3\beta_0 r_3 \delta_3 + 3\beta_0^2 r_2 \delta_2^2 \\ &- \beta_0^3 r_1 \delta_1^3 + \frac{5}{2}\beta_1 \beta_0 r_1 \delta_1^2] a(Q)^4 + \mathcal{O}(a^5)\end{aligned} \tag{20}$$

Shows the general way that nonconformal terms enter an observable

$$a_{\mathcal{S}} = \frac{\alpha_{\mathcal{S}}}{4\pi} = \frac{g_{\mathcal{S}}^2}{16\pi^2}$$

$$\frac{\mathrm{d}a_{\mathcal{S}}}{\mathrm{d}\ln\mu^2} = \beta_{\mathcal{S}}(a) = -a^2[\beta_0 + \beta_1 a + \beta_2^{\mathcal{S}} a^2 + \beta_3^{\mathcal{S}} a^3 + \cdots]$$

$$\beta_0 = \frac{11}{3}N_C - \frac{2}{3}n_F$$

QED: N_C=0

Relating different renormalization scales:

Taylor expanding $a(\mu)$ around $\ln(\mu_0)$:

$$a(\mu) = a(\mu_0) - \beta_0 a(\mu_0)^2 \ln\frac{\mu^2}{\mu_0^2} - \left[\beta_1 - \beta_0^2 \ln\frac{\mu^2}{\mu_0^2}\right] a(\mu_0)^3 \ln\frac{\mu^2}{\mu_0^2} + \cdots$$

General pattern of pQCD

Set multiple renormalization scales -- Lensing, DGLAP, ERBL Evolution ...

$Choose\ renormalization\ scheme;\ e.g.\ \alpha_s^R(\mu_R^{\rm init})$

↓

$Choose\ \mu_R^{init};\ arbitrary\ initial\ renormalization\ scale$

↓

$Identify\ \{\beta_i^R\}-terms\ using$ δ- $terms$

$through\ the\ PMC-BLM\ correspondence\ principle$

↓ order by order

$Shift\ scale\ of\ \alpha_s\ to\ \mu_R^{\rm PMC}\ to\ eliminate\ \{\beta_i^R\}-terms$

↓

$Conformal\ Series$

$Result\ is\ independent\ of\ \mu_R^{\rm init}\ and\ scheme\ at\ fixed\ order$

PMC/BLM

No renormalization scale ambiguity!

Result is independent of Renormalization scheme and initial scale!

QED Scale Setting at N_C=0

Eliminates unnecessary systematic uncertainty

Scale fixed at each order

δ-Scheme automatically identifies β-terms!

Principle of Maximum Conformality

Xing-Gang Wu, Matin Mojaza Leonardo di Giustino, SJB

A robot can compute the PMC scales

BLM/PMC Scale-Setting for $R(Q)$

$$R_{e^+e^-}(Q) = 3\sum_q e_q^2 \Big[1 + \left(a^{\overline{MS}}(Q)\right) + (1.9857 - 0.1152 n_f)\left(a^{\overline{MS}}(Q)\right)^2$$
$$+ \left(-6.63694 - 1.20013 n_f - 0.00518 n_f^2 - 1.240 \frac{(\sum_q e_q)^2}{3\sum_q e_q^2}\right)\left(a^{\overline{MS}}(Q)\right)^3$$
$$+ \left(-156.61 + 18.77 n_f - 0.7974 n_f^2 + 0.0215 n_f^3 + C \frac{(\sum_q e_q)^2}{3\sum_q e_q^2}\right)\left(a^{\overline{MS}}(Q)\right)^4 \Big]$$

$$\frac{\sigma(e^+e^- \to \text{hadrons}, Q)}{\sigma(e^+e^- \to \mu^+\mu^-, Q)} \equiv R(Q)$$

C is for singlet contribution and is small

As usual, we set C=0

P.A. Baikov, K.G. Chetyrkin and J.H. Kuhn, Phys.Rev.Lett.**101**, 012002(2008); arXiv:0906.2987[hep-ph]; K. Nakamura et al. (Particle Data Group), J.Phys. G**37**, 075021 (2010).

$$R_{e^+e^-}(Q) = 3\sum_q e_q^2 \Big[1 + \left(a_s^{\overline{MS}}(Q^*)\right) + \widetilde{A}\left(a_s^{\overline{MS}}(Q^{**})\right)^2$$
$$+ \widetilde{\widetilde{B}}\left(a_s^{\overline{MS}}(Q^{***})\right)^3 + \widetilde{\widetilde{\widetilde{C}}}\left(a_s^{\overline{MS}}(Q^{***})\right)^4 \Big]$$

$$\bar{R}_{e^+e^-}(s) = \frac{1}{2\pi i}\int_{-s-i\epsilon}^{-s+i\epsilon} \frac{\bar{D}(Q^2)}{Q^2} dQ^2 \qquad \bar{D}(Q^2) = \gamma(a) - \beta(a)\frac{d}{da}\Pi(Q^2,a)$$

Initial expression

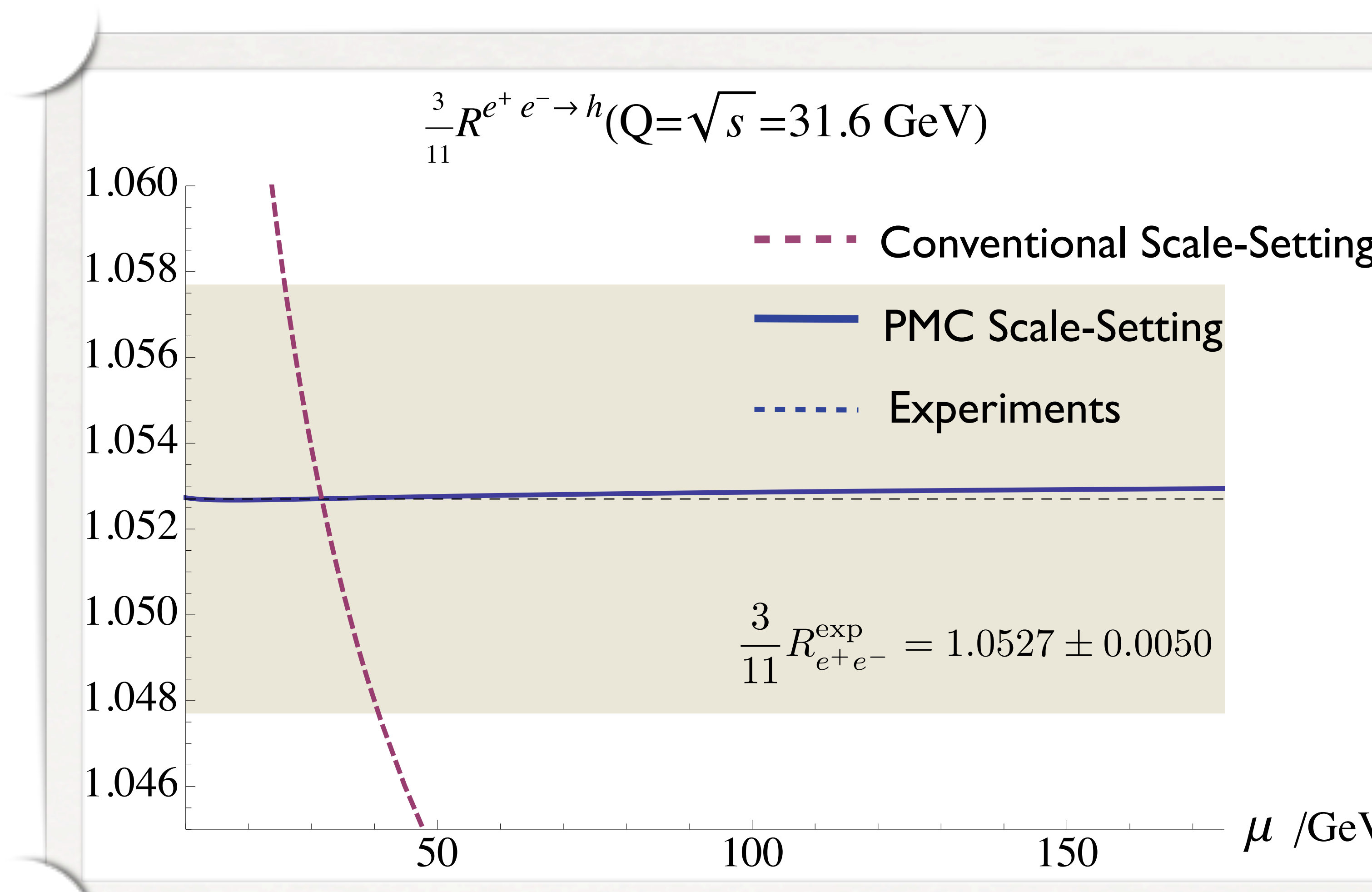
$$\bar{R}_{e^+e^-}(s) = \gamma_0 + \gamma_1 a(\mu) + [\gamma_2 + \beta_0 \underline{\Pi_1}]a(\mu)^2 + [\gamma_3 + \beta_1 \underline{\Pi_1} + 2\beta_0 \underline{\Pi_2} - \beta_0^2 \underline{\frac{\pi^2\gamma_1}{3}}]a(\mu)^3$$
$$+ [\gamma_4 + \beta_2 \underline{\Pi_1} + 2\beta_1 \underline{\Pi_2} + \beta_0 \Pi_3 - \frac{5}{2}\beta_0\beta_1 \underline{\frac{\pi^2\gamma_1}{3}} - 3\beta_0^2 \frac{\pi^2\gamma_2}{3} - \beta_0^3 \pi^2 \Pi_1]a(\mu)^4$$

Final expression

$$\bar{R}_{e^+e^-}(Q) = \gamma_0 + \gamma_1 a(Q_1) + \gamma_2 a(Q_2)^2 + \gamma_3 a(Q_3)^3 + \gamma_4 a(Q_4)^4$$

Final PMC Scales

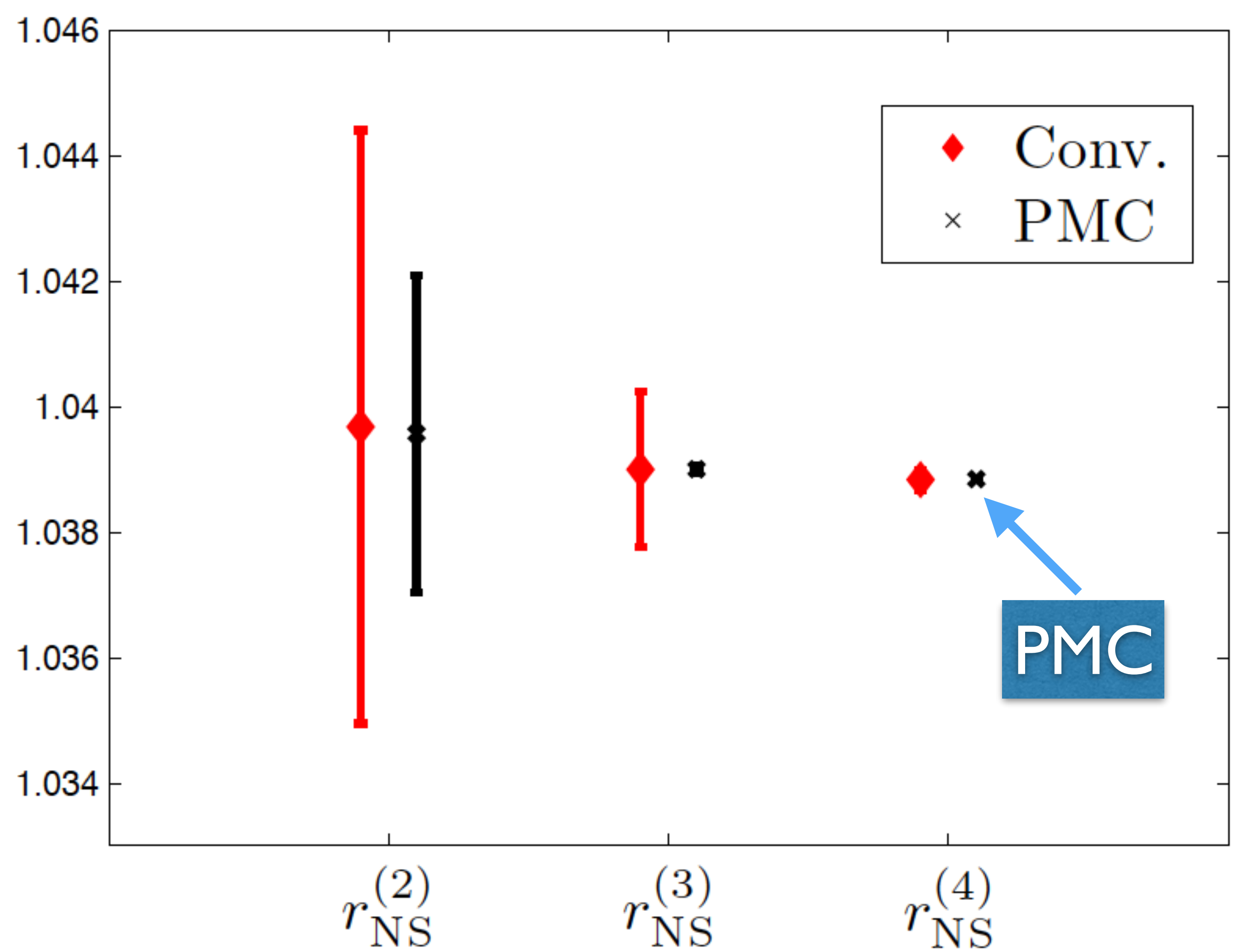
$$Q_1 = 1.3\ Q,\ Q_2 = 1.2\ Q,$$
$$Q_3 = 5.3\ Q,\ Q_4 \sim Q$$



Reanalysis of the Higher Order Perturbative QCD corrections to Hadronic Z Decays using the Principle of Maximum Conformality

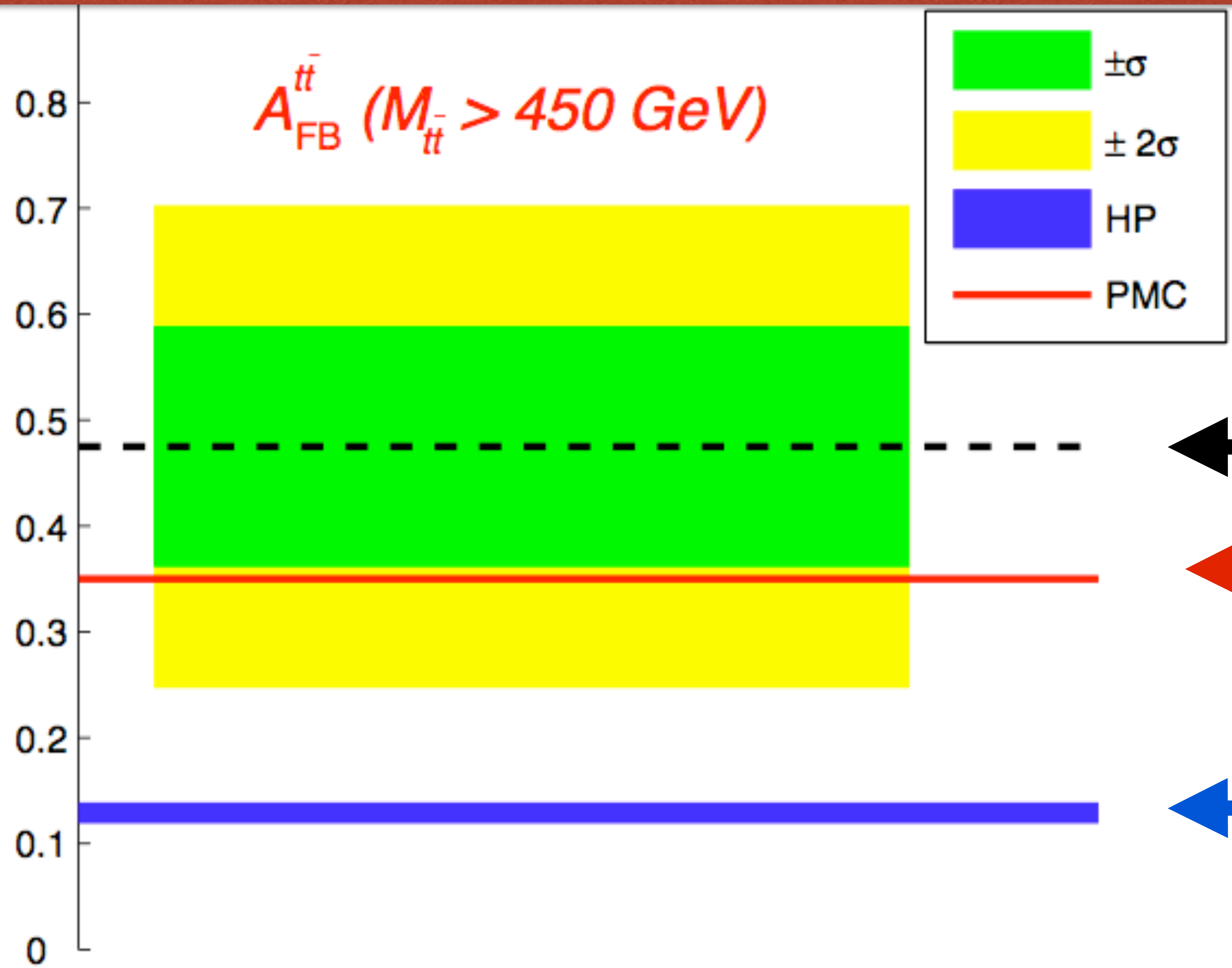
S-Q Wang, X-G Wu, sjb

P.A. Baikov, K.G. Chetyrkin, J.H. Kuhn, and J. Rittinger, Phys. Rev. Lett. 108, 222003 (2012).



The values of $r_{\rm NS}^{(n)} = 1 + \sum_{i=1}^{n} C_i^{\rm NS} a_s^i$ and their errors $\pm|C_n^{\rm NS} a_s^n|_{\rm MAX}$. The diamonds and the crosses are for conventional (Conv.) and PMC scale settings, respectively. The central values assume the initial scale choice $\mu_r^{\rm init} = M_Z$.

The Renormalization Scale Ambiguity for Top-Pair Production Asymmetry at the Tevatron is Eliminated Using the 'Principle of Maximum Conformality' (PMC)



Xing-Gang Wu
S.-Q. Wang
Z.-G. Shi
SJB

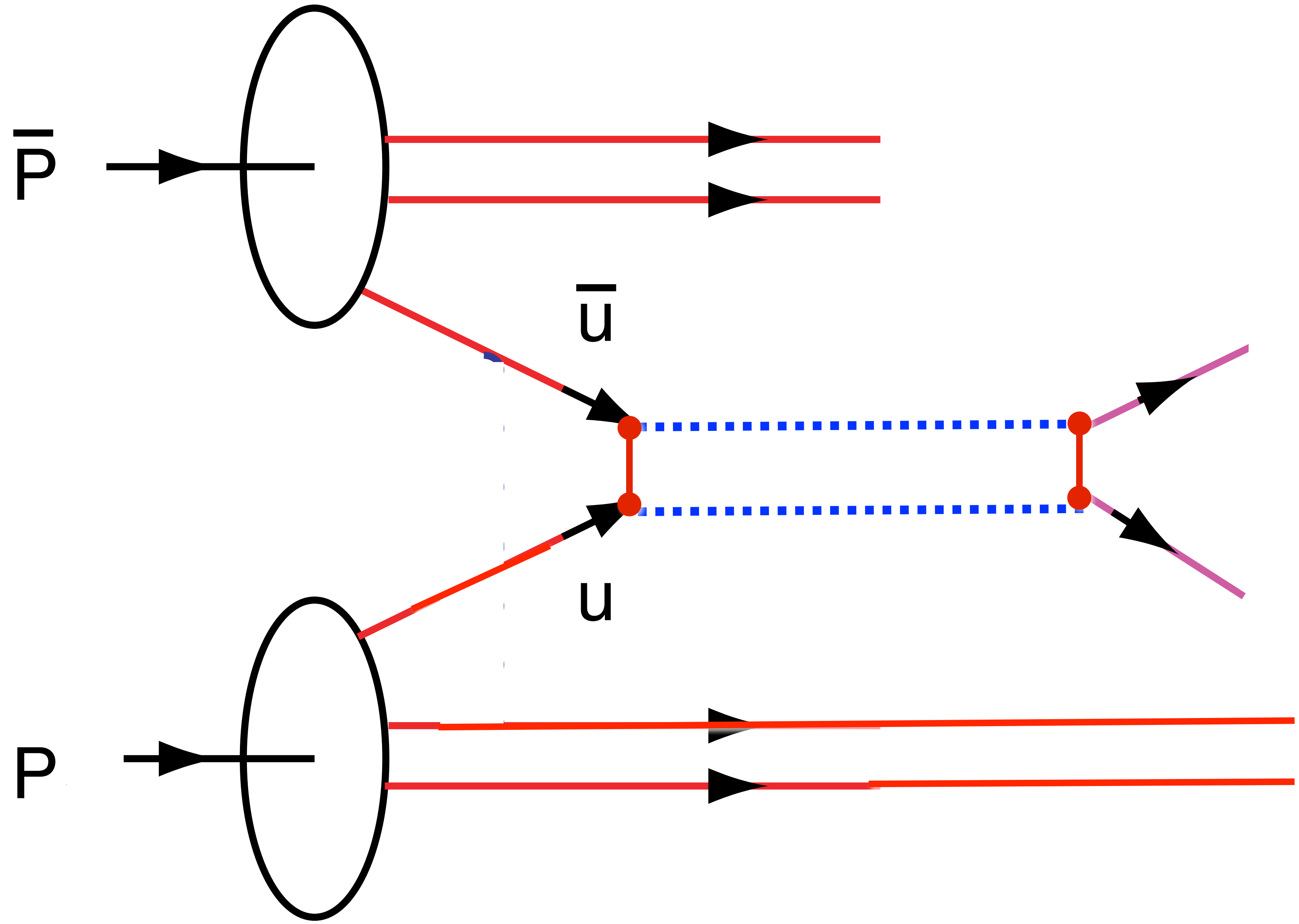
Experimental asymmetry

PMC Prediction

Conventional guess for renormalization scale and range

Top quark forward-backward asymmetry predicted by pQCD NNLO within 1 σ of CDF/D0 measurements using PMC/BLM scale setting

Implications for the $\bar{p}p \to t\bar{t}X$ asymmetry at the Tevatron



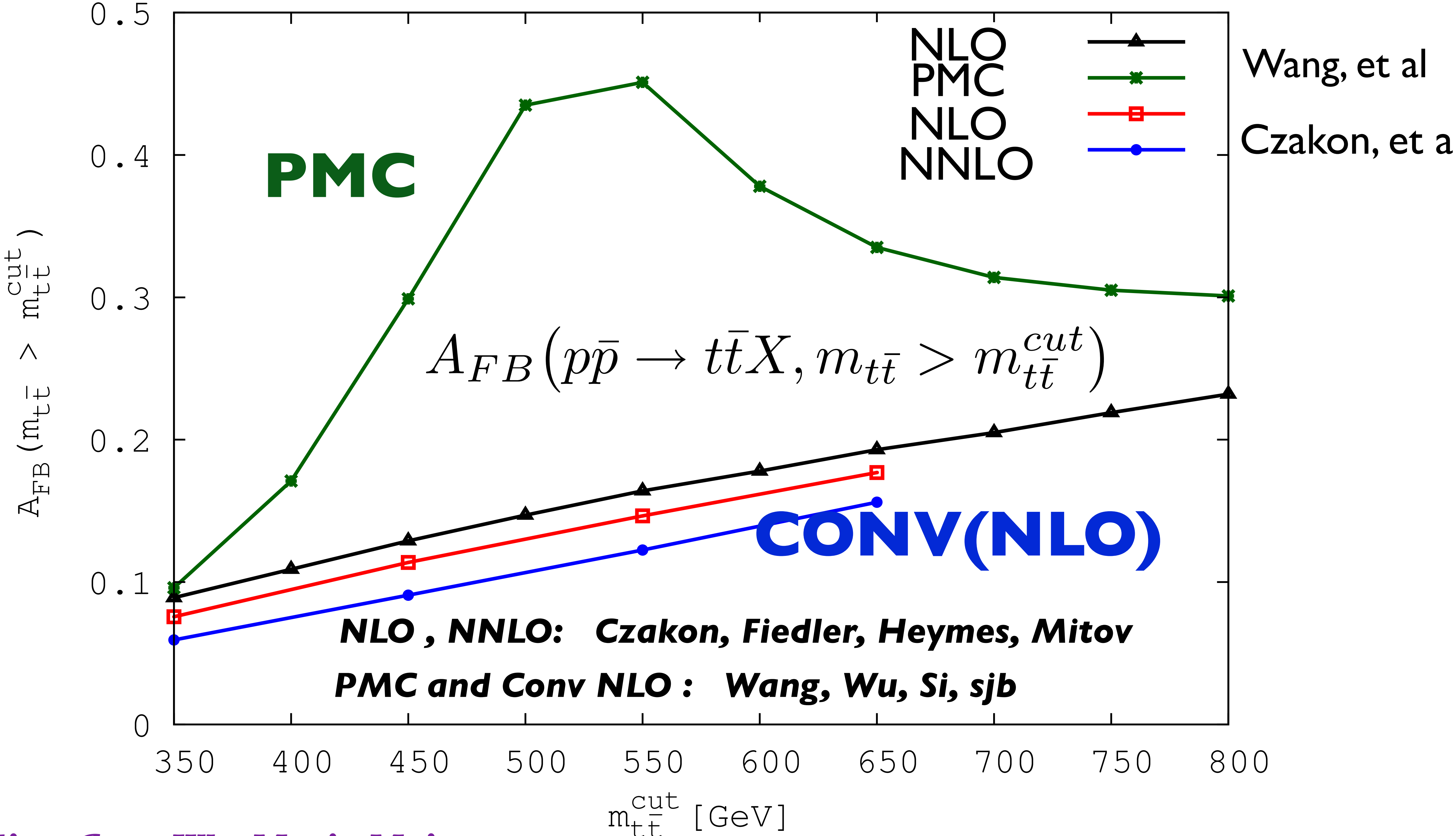
Interferes with Born term.

Small value of renormalization scale increases asymmetry, just as in QED

Xing-Gang Wu, sjb

NNLO QCD predictions for fully-differential top-quark pair production at the Tevatron arXiv:1601.05375

Michał Czakon,[a] Paul Fiedler,[a] David Heymes[b] and Alexander Mitov[b]



Xing-Gang Wu, Matin Mojaza
Leonardo di Giustino, SJB

Predictions for the cumulative front-back asymmetry.

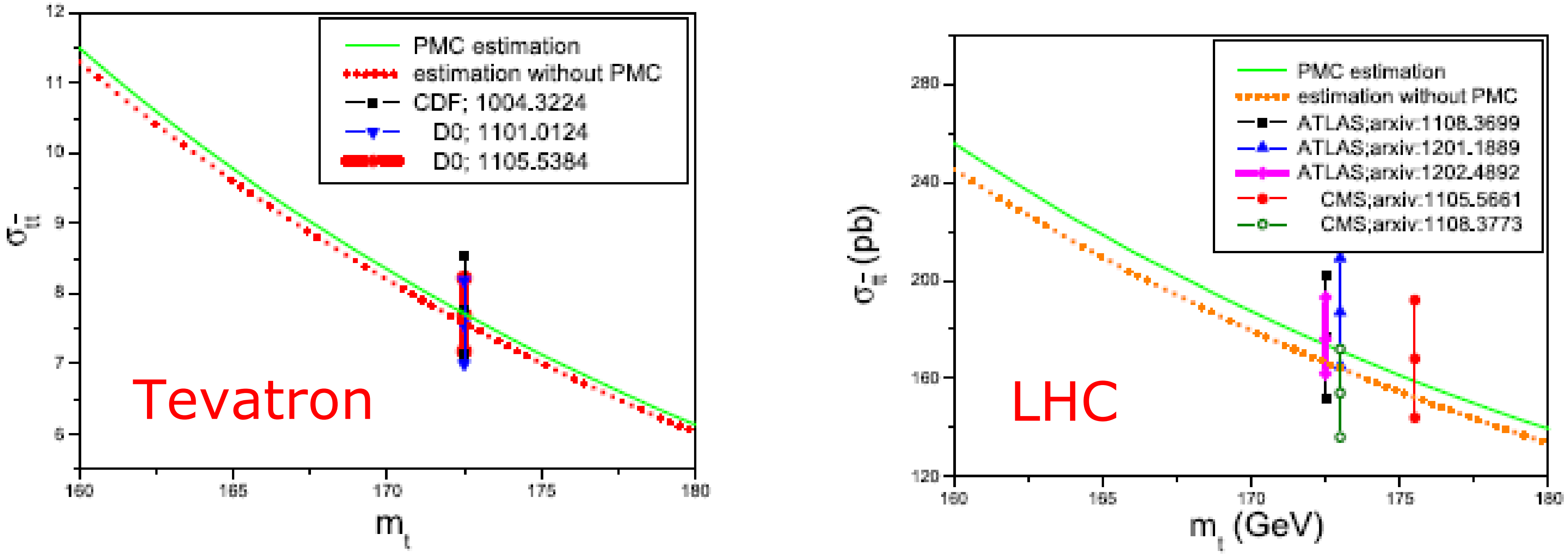
- *Application of the Principle of Maximum Conformality to the Top-Quark Charge Asymmetry at the LHC*

Sheng-Quan Wang, Xing-Gang Wu (Chongqing U. & Beijing, Inst. Theor. Phys.), Zong-Guo Si (Shandong U.), Stanley J. Brodsky (SLAC). Oct 6, 2014. 10 pp.

Published in **Phys.Rev. D90 (2014) 11, 114034**
SLAC-PUB-16116
DOI: 10.1103/PhysRevD.90.114034
e-Print: **arXiv:1410.1607 [hep-ph]**

Conventional scale choice: m_t
"Lucky guess" for total rate

X-G Wu, sjb



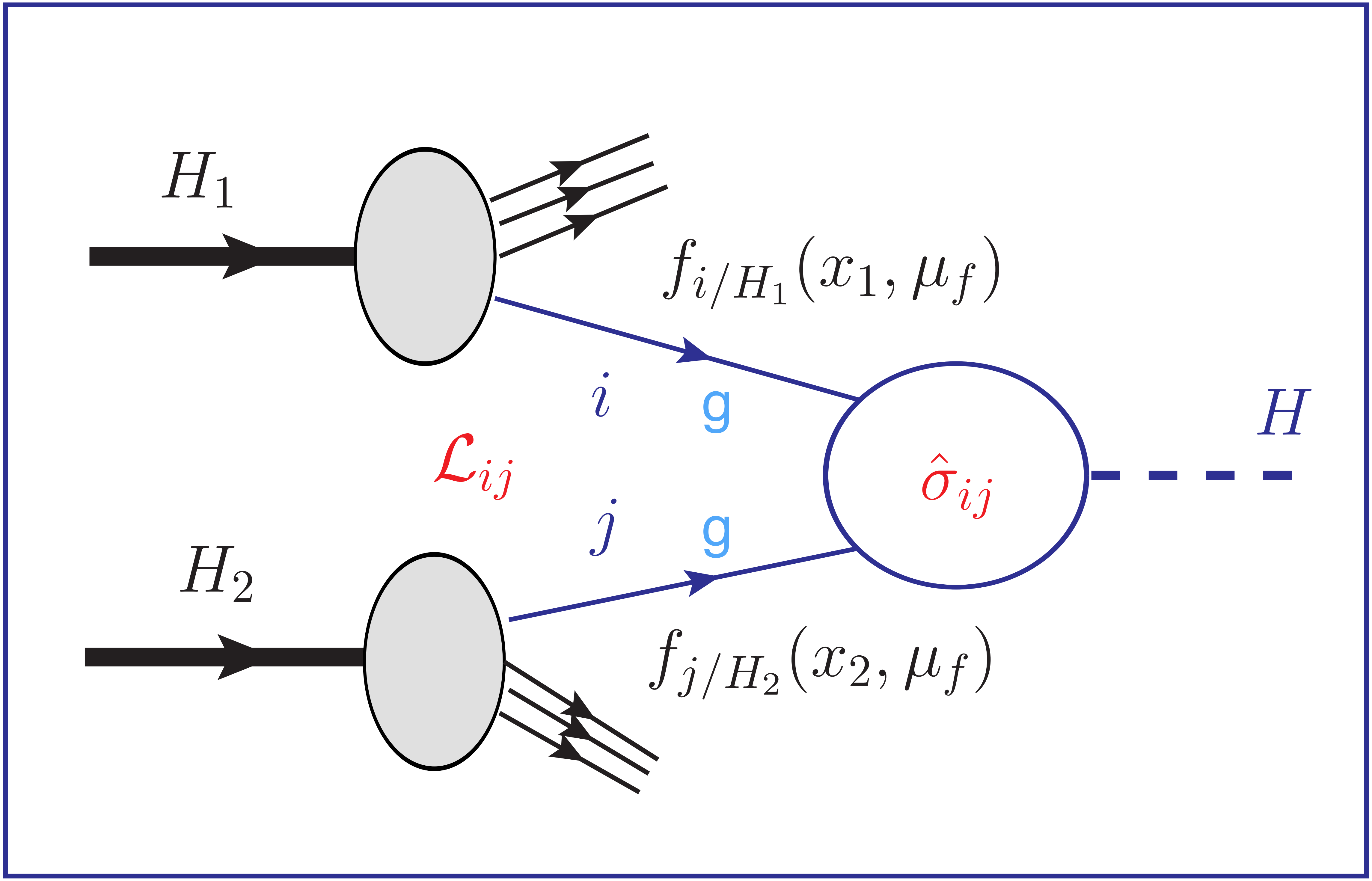
$$m_t = 172.9 \pm 1.1\ \mathrm{GeV}$$

$$\sigma_{\mathrm{Tevatron,\ 1.96\ TeV}} = 7.626^{+0.265}_{-0.257}\ \mathrm{pb}$$
$$\sigma_{\mathrm{LHC,\ 7\ TeV}} = 171.8^{+5.8}_{-5.6}\ \mathrm{pb}$$
$$\sigma_{\mathrm{LHC,\ 14\ TeV}} = 941.3^{+28.4}_{-26.5}\ \mathrm{pb}$$

PDF+α_s error

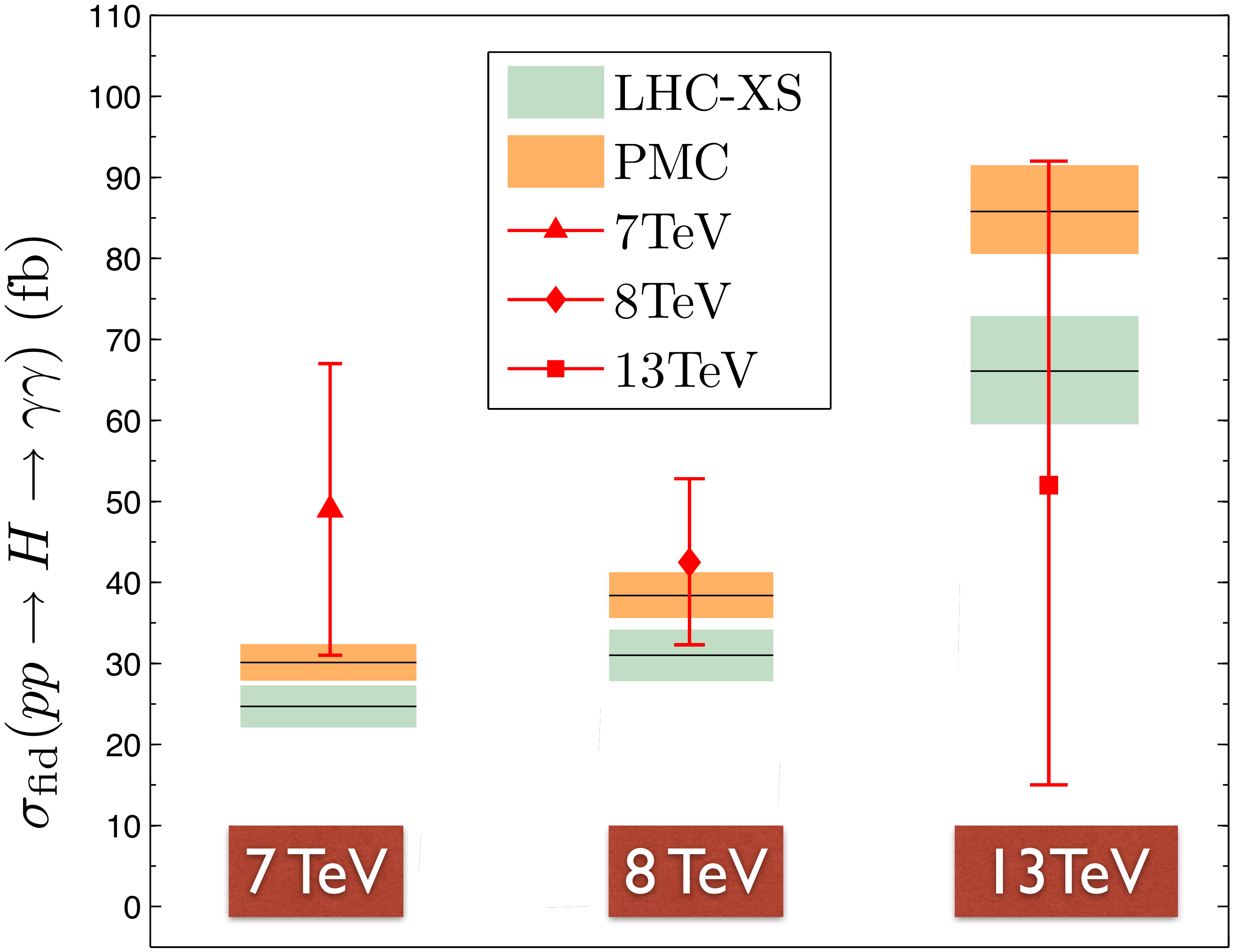
$$\alpha_s(m_z) = 0.118 \pm 0.001$$

$$\sigma_{\mathrm{Tevatron,\ 1.96\ TeV}} = 7.626^{+0.143}_{-0.130}\ \mathrm{pb}$$
$$\sigma_{\mathrm{LHC,\ 7\ TeV}} = 171.8^{+3.8}_{-3.5}\ \mathrm{pb}$$
$$\sigma_{\mathrm{LHC,\ 14\ TeV}} = 941.3^{+14.6}_{-15.6}\ \mathrm{pb}$$


H_1
$f_{i/H_1}(x_1, \mu_f)$
i
g
$\mathcal{L}_{ij}$
$\hat{\sigma}_{ij}$
H
j
g
H_2
$f_{j/H_2}(x_2, \mu_f)$

S-Q Wang, X-G Wu, sjb

Preliminary



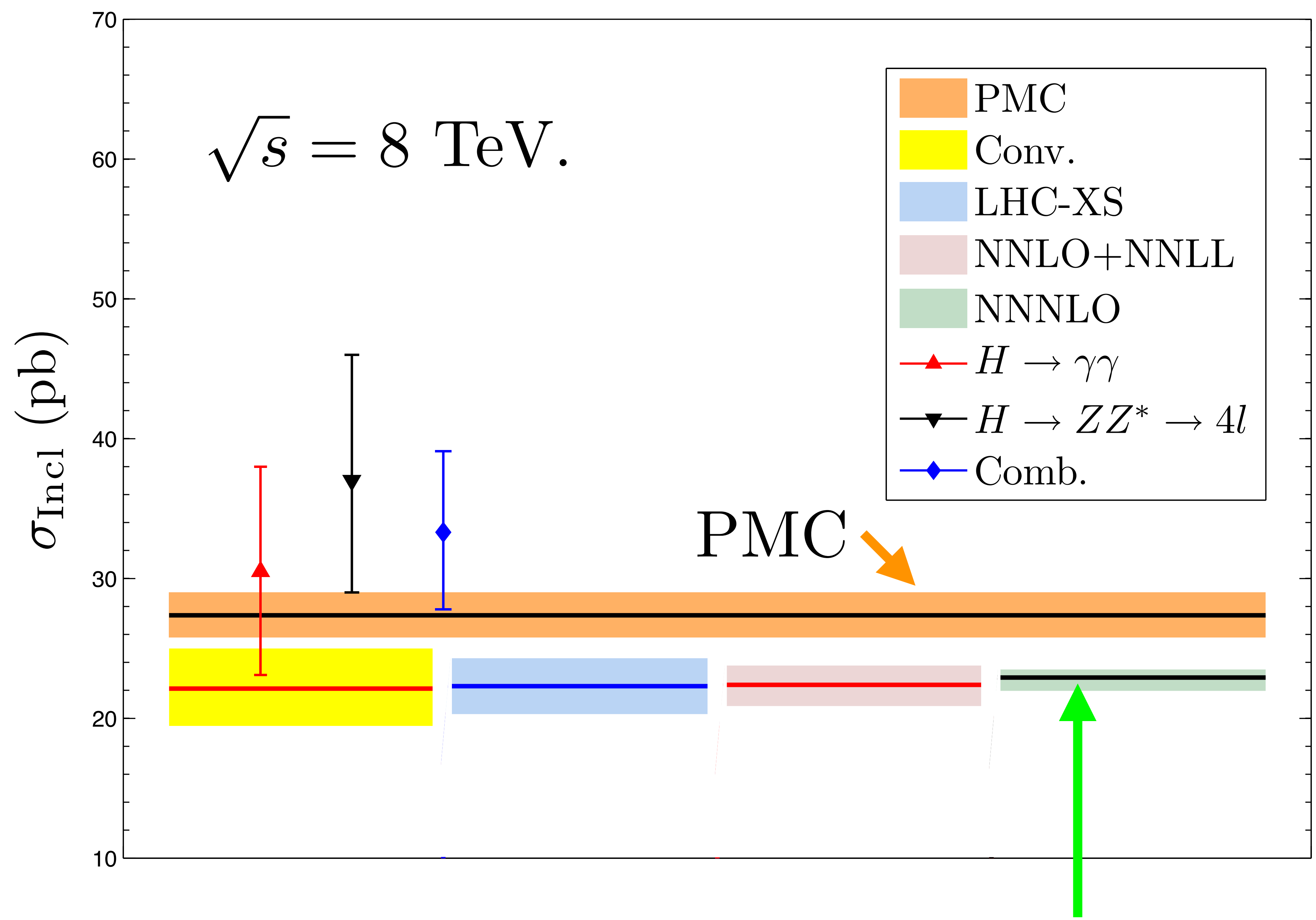
Comparison of the PMC predictions for the fiducial cross section $\sigma_{\rm fid}(pp \to H \to \gamma\gamma)$ with the ATLAS measurements at various collision energies. The LHC-XS predictions are presented as a comparison.

$\sigma_{\rm fid}(pp \to H \to \gamma\gamma)$	7 TeV	8 TeV	13 TeV
ATLAS data [48]	49 ± 18	$42.5^{+10.3}_{-10.2}$	52^{+40}_{-37}
LHC-XS [3]	24.7 ± 2.6	31.0 ± 3.2	$66.1^{+6.8}_{-6.6}$
PMC prediction	$30.1^{+2.3}_{-2.2}$	$38.4^{+2.9}_{-2.8}$	$85.8^{+5.7}_{-5.3}$

S-Q Wang, X-G Wu, sjb

Preliminary

$$\sigma^{gg}(pp \to HX)$$



NNNLO (conventional)

S-Q Wang, X-G Wu, sjb

Preliminary

$\sqrt{S}$	7 TeV	8 TeV	13 TeV
ATLAS($H \to \gamma\gamma$) [4]	35^{+13}_{-12}	$30.5^{+7.5}_{-7.4}$	40^{+31}_{-28}
ATLAS($H \to ZZ^* \to 4l$) [4]	33^{+21}_{-16}	37^{+9}_{-8}	12^{+25}_{-16}
LHC-XS [3]	17.5 ± 1.6	22.3 ± 2.0	$50.9^{+4.5}_{-4.4}$
PMC predictions	$21.21^{+1.36}_{-1.32}$	$27.44^{+1.65}_{-1.59}$	$65.72^{+3.46}_{-3.01}$

TABLE IV: Total inclusive cross-sections (in unit: pb) for the Higgs production at the LHC with the collision energies $\sqrt{S} = 7$, 8 and 13 TeV, respectively. The inclusive cross-section $\sigma_{\rm Incl} = \sigma_{\rm ggH} + \sigma_{\rm xH} + \sigma_{\rm EW}$.

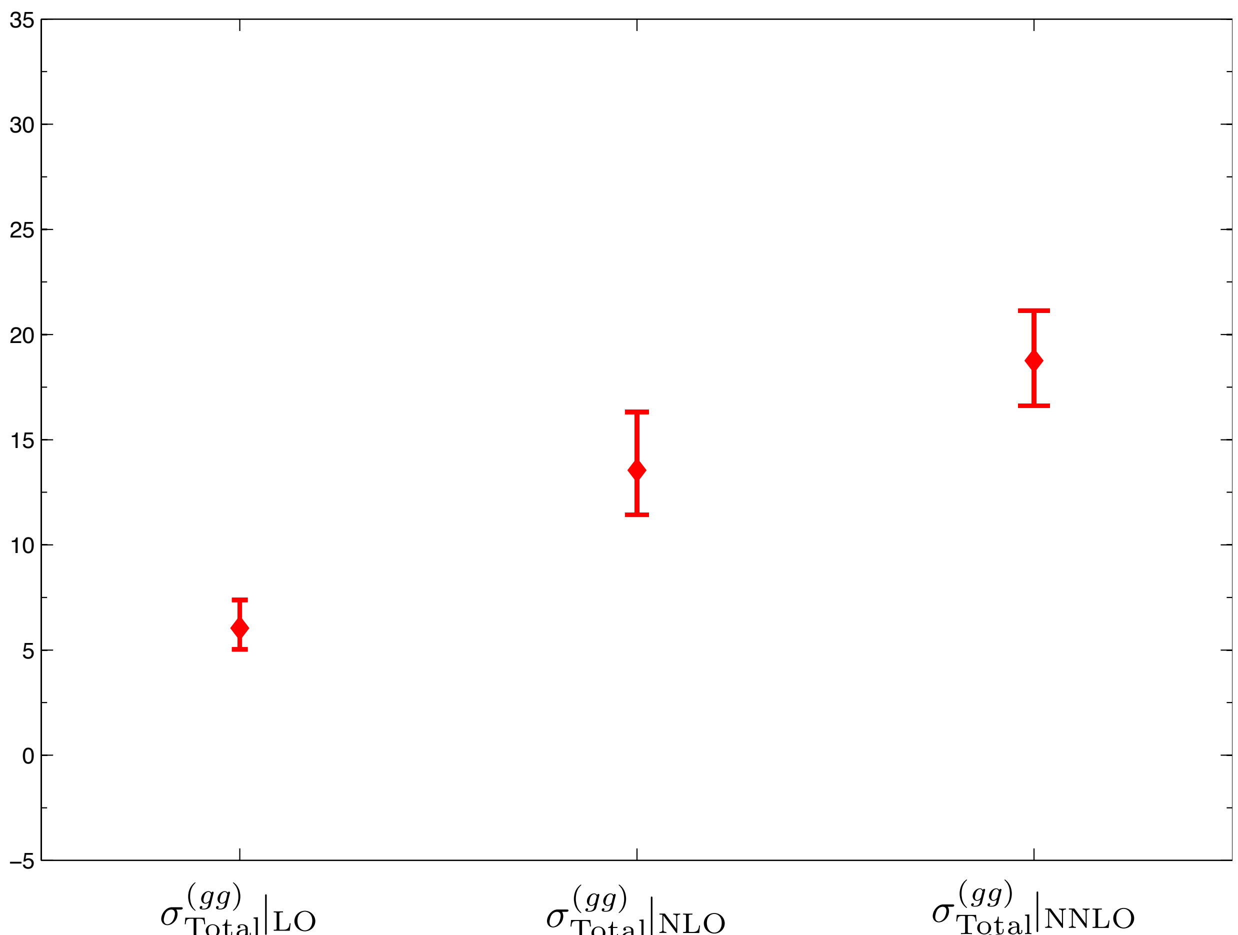
S-Q Wang, X-G Wu, sjb

Preliminary

$$\sigma^{gg}(pp \to HX) \text{ in pb}$$

Conventional

Uncertainty: Vary Initial scale

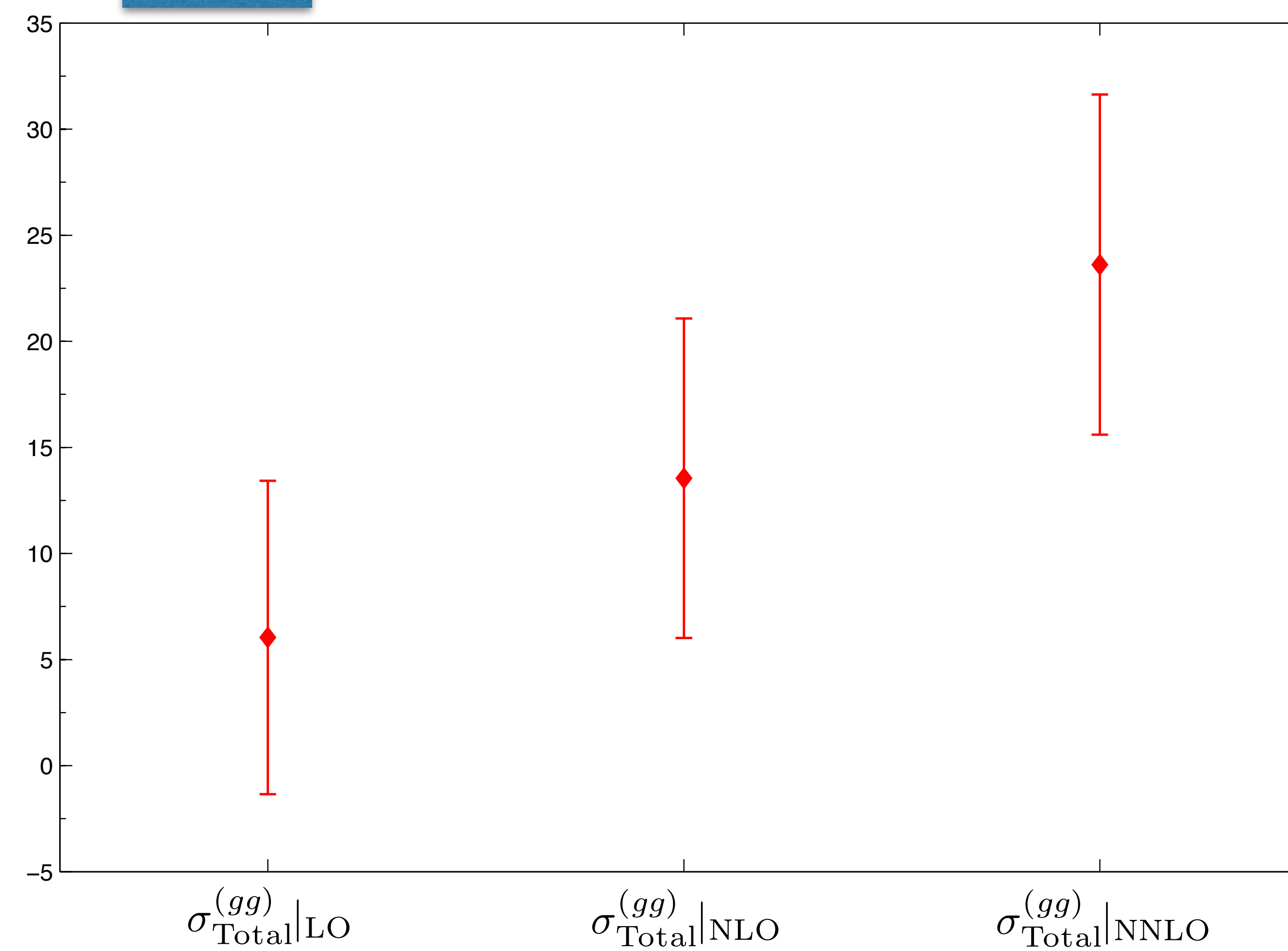


Results for gluon-fusion total cross-section $\sigma^{(gg)}_{\rm Total}|_n = \sum_{i={\rm LO}}^{n} \mathcal{C}_i\, a_s^{i+1}$ under conventional scale-setting, where n stands for LO, NLO or NNLO, respectively. The error bars stand for the predictions of "uncalculated" higher-order terms, which are obtained by varying $\mu_r \in [m_H/2, 2m_H]$ in all "known" low-order terms.

Error is underestimated using variation of guessed scale

PMC

Uncertainty: Uncalculated Higher Order Terms



$i={\rm LO}$

der PMC scale-setting, where n stands for LO, NLO or NNLO, respectively. The error bar for $i_{\rm th}$-order stands for the prediction of "uncalculated" higher-order terms, which is taken as $\pm|\tilde{\mathcal{C}}_i\, a_s^{i+1}(Q_i^{gg})|_{\rm MAX}$.

PMC: Conservative Error Estimate

Preliminary

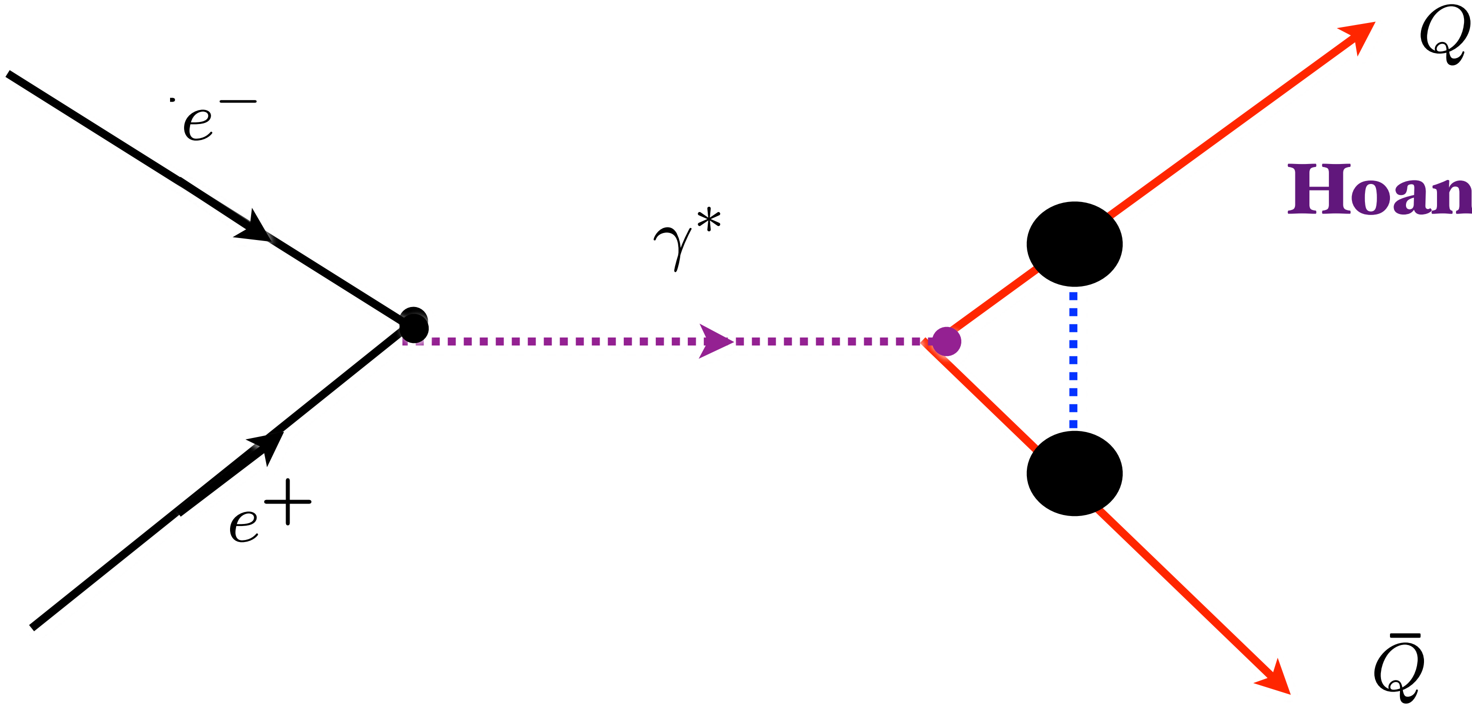
S-Q Wang, X-G Wu, sjb

$$\sigma^{gg}(pp \to HX)$$ in pb

	Conventional				PMC			
μ_r	LO	NLO	$\mathrm{N^2LO}$	Total	LO	NLO	$\mathrm{N^2LO}$	Total
$m_H/4$	9.42	10.64	3.50	23.56	6.02	9.58	8.01	23.61
$m_H/2$	7.43	8.89	4.82	21.14	6.02	9.58	8.01	23.61
m_H	6.02	7.53	5.21	18.76	6.02	9.58	8.01	23.61
$2m_H$	4.98	6.45	5.19	16.62	6.02	9.58	8.01	23.61
$4m_H$	4.19	5.58	4.95	14.35	6.02	9.58	8.01	23.61

The gluon-fusion cross-section $\sigma_m^{(gg)}$ (in unit: pb) using the conventional and PMC scale-settings at $\sqrt{s} = 8$ TeV, where five typical initial scales $\mu_r = m_H/4$, $m_H/2$, m_H, $2m_H$, $4m_H$ are adopted. $\mu_f = m_H$.

Insensitivity of PMC predictions to choice of initial scale



$$F_1 + F_2 = [1 - 2\frac{\alpha_s(se^{3/4}/4)}{\pi}] \times [1 + \frac{\pi\alpha_s(sv^2)}{4v}]$$

Angular distributions of massive quarks close to threshold.

Example of Multiple BLM Scales

Need QCD coupling at small scales at low relative velocity v

Angular distributions of massive quarks and leptons close to threshold.

Single-spin asymmetries

Leading Twist Sivers Effect

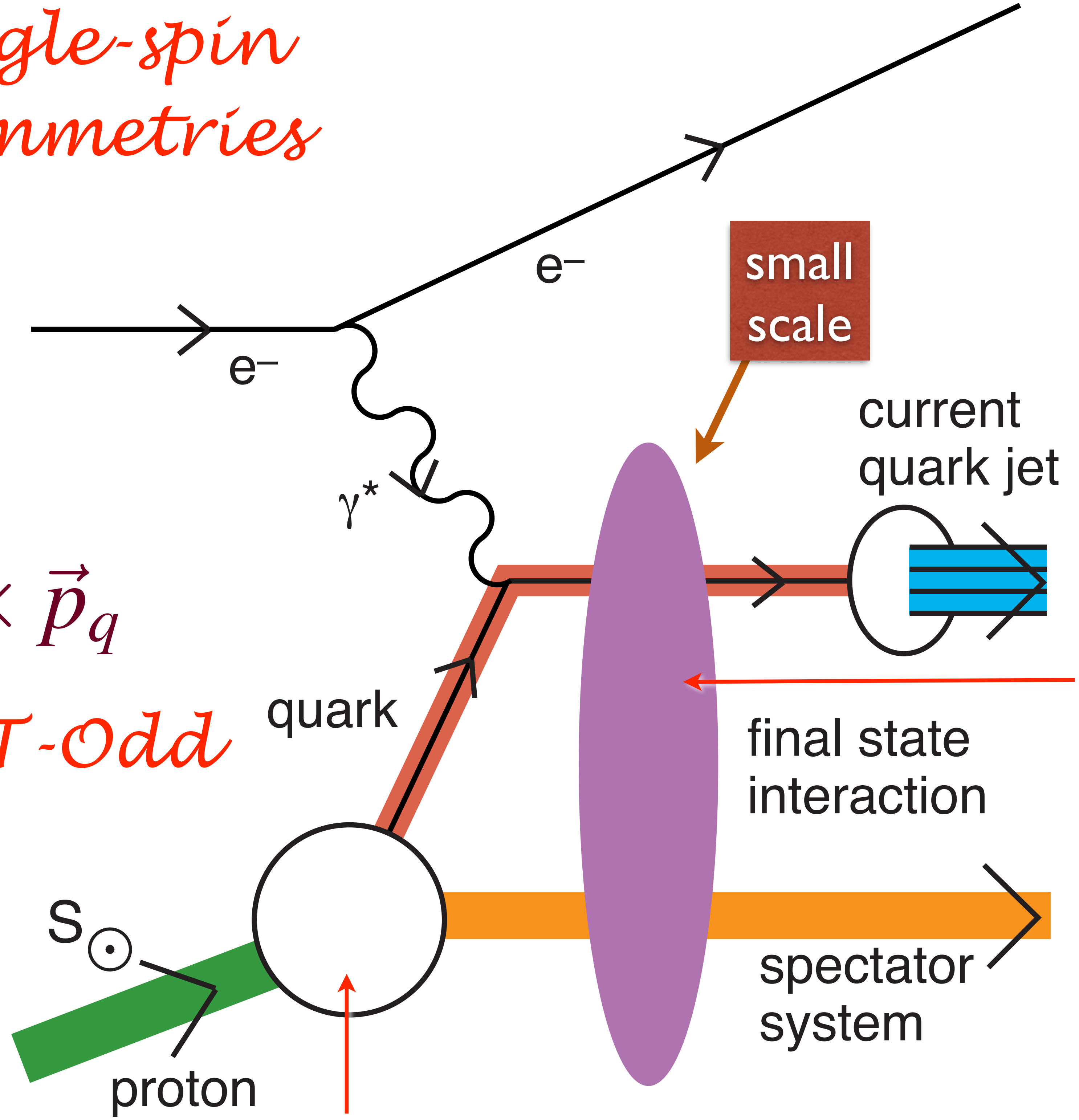
Hwang, Schmidt, sjb

Collins, Burkardt, Ji, Yuan. Pasquini, ...

$$i\ \vec{S}_p \cdot \vec{q} \times \vec{p}_q$$

Pseudo- T-Odd

"Lensing" involves soft scales



Light-Front Wavefunction S and P- Waves!

QCD S- and P- Coulomb Phases --Wilson Line

"Lensing Effect"

Leading-Twist Rescattering Violates pQCD Factorization!

Sign reversal in DY!

Double Initial-State Interactions generate anomalous $\cos 2\phi$

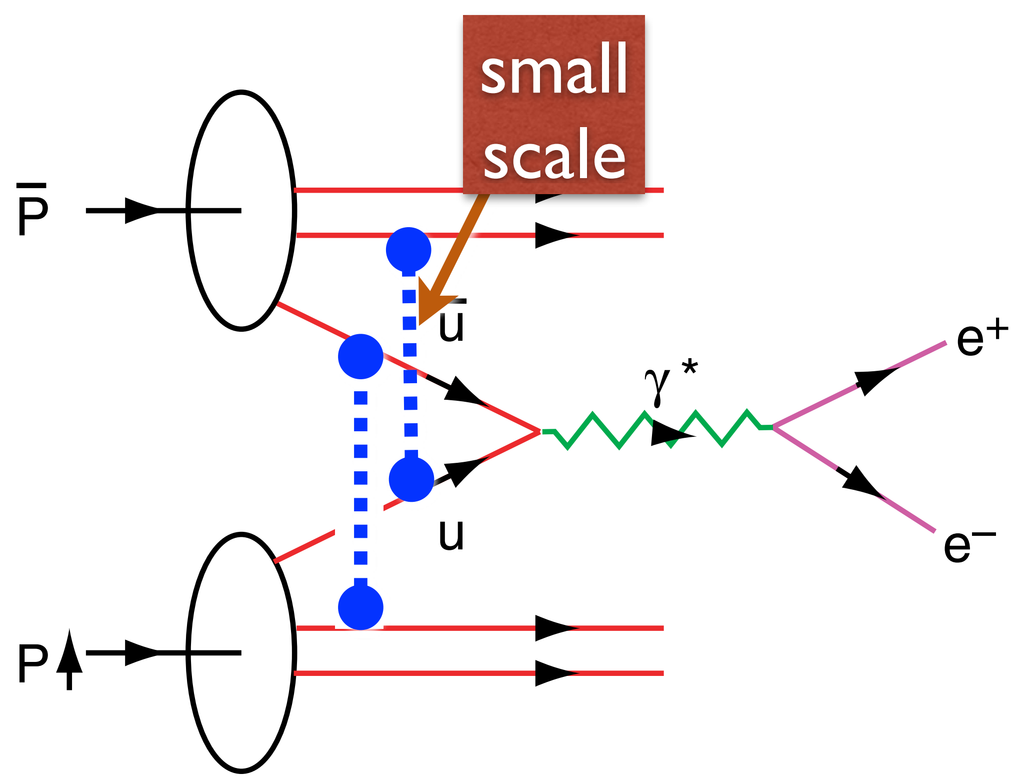
Boer, Hwang, sjb

Drell-Yan planar correlations

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} \propto \left(1 + \lambda \cos^2\theta + \mu \sin 2\theta \, \cos\phi + \frac{\nu}{2} \sin^2\theta \cos 2\phi\right)$$

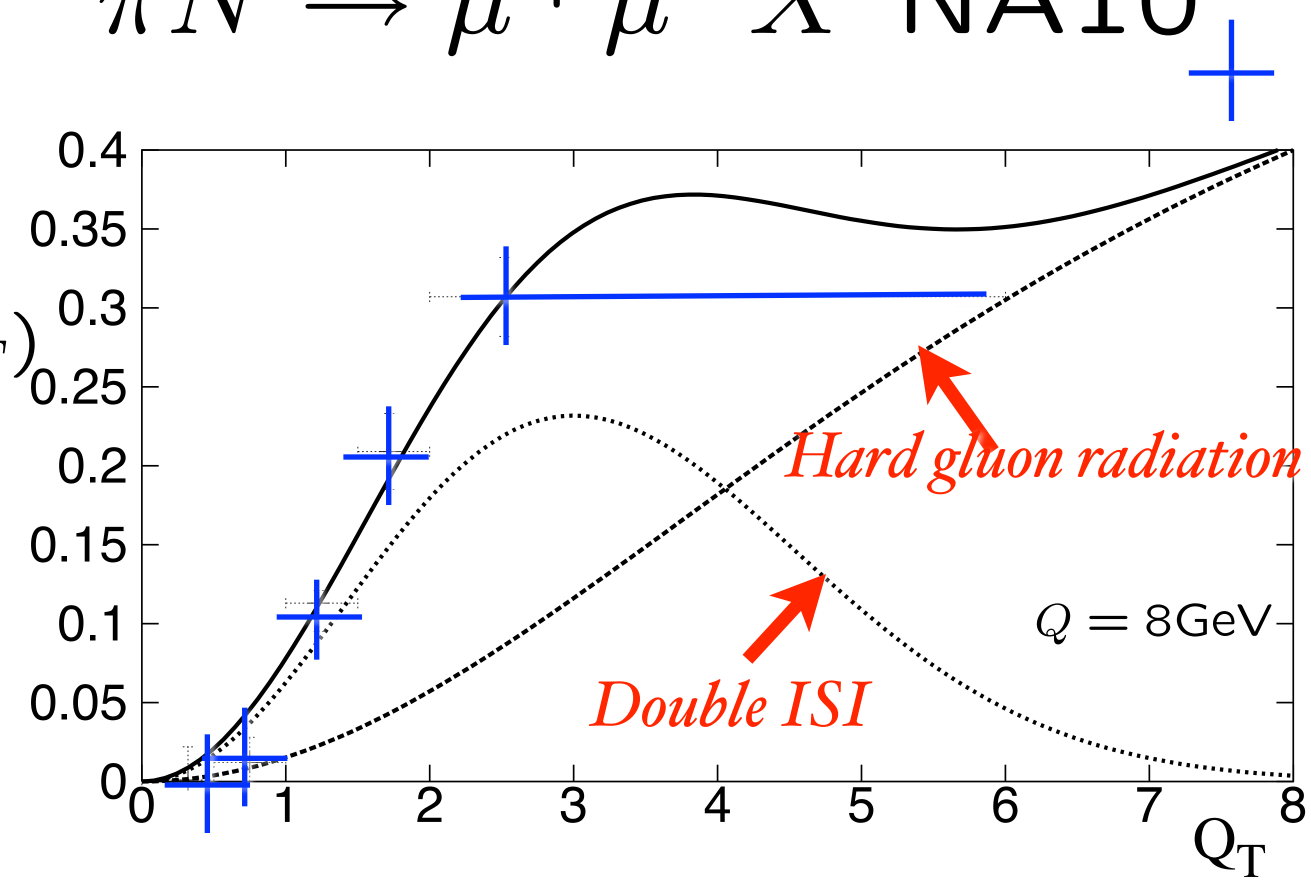
PQCD Factorization (Lam Tung): $1 - \lambda - 2\nu = 0$

$$\frac{\nu}{2} \propto h_1^{\perp}(\pi)\, h_1^{\perp}(N)$$



Violates Lam-Tung relation!

$\pi N \to \mu^+\mu^- X$ NA10



Myths concerning scale setting

- Renormalization scale "unphysical": No optimal physical scale

- Can ignore possibility of multiple physical scales

- Accuracy of PQCD prediction can be judged by taking arbitrary guess $\mu_R = Q$ with an arbitrary range $Q/2 < \mu_R < 2Q$

- Factorization scale should be taken equal to renormalization scale $\mu_F = \mu_R$

These assumptions are untrue in QED and thus they cannot be true for QCD

Clearly heuristic. Wrong in QED. Scheme dependent!

Essential Points

- *Physical Results cannot depend on choice of Scheme*
- *Different PMC scales at each order*
- *No scale ambiguity!*
- *Series identical to conformal theory*
- *Relation between observables scheme independent, transitive*
- *Choice of initial scale irrelevant even at finite order*
- *Identify β terms using R_δ method*

Relate Observables to Each Other

- Eliminate intermediate scheme
- No scale ambiguity
- Transitive!
- Commensurate Scale Relations
- Conformal Template
- Example: Generalized Crewther Relation

$$R_{e^+e^-}(Q^2) \equiv 3 \sum_{\text{flavors}} e_q{}^2 \left[1 + \frac{\alpha_R(Q)}{\pi}\right].$$

$$\int_0^1 dx\, \left[g_1^{ep}(x,Q^2) - g_1^{en}(x,Q^2)\right] \equiv \frac{1}{3}\left|\frac{g_A}{g_V}\right|\left[1 - \frac{\alpha_{g_1}(Q)}{\pi}\right]$$

$$R_{e^+e^-}(Q^2) \equiv 3 \sum_{\text{flavors}} e_q^{\,2} \left[1 + \frac{\alpha_R(Q)}{\pi} \right].$$

$$\int_0^1 dx \left[g_1^{ep}(x, Q^2) - g_1^{en}(x, Q^2) \right] \equiv \frac{1}{3} \left| \frac{g_A}{g_V} \right| \left[1 - \frac{\alpha_{g_1}(Q)}{\pi} \right]$$

$$\frac{\alpha_{g_1}(Q)}{\pi} = \frac{\alpha_R(Q^*)}{\pi} - \left(\frac{\alpha_R(Q^{**})}{\pi} \right)^2 + \left(\frac{\alpha_R(Q^{***})}{\pi} \right)^3$$

Geometric Series in Conformal QCD

Generalized Crewther Relation

Lu, Kataev, Gabadadze, Sjb

$$\frac{\alpha_R(Q)}{\pi} = \frac{\alpha_{\overline{\rm MS}}(Q)}{\pi} + \left(\frac{\alpha_{\overline{\rm MS}}(Q)}{\pi}\right)^2 \left[\left(\frac{41}{8} - \frac{11}{3}\zeta_3\right) C_A - \frac{1}{8}C_F + \left(-\frac{11}{12} + \frac{2}{3}\zeta_3\right) f\right]$$
$$+ \left(\frac{\alpha_{\overline{\rm MS}}(Q)}{\pi}\right)^3 \left\{\left(\frac{90445}{2592} - \frac{2737}{108}\zeta_3 - \frac{55}{18}\zeta_5 - \frac{121}{432}\pi^2\right) C_A^2 + \left(-\frac{127}{48} - \frac{143}{12}\zeta_3 + \frac{55}{3}\zeta_5\right) C_A C_F - \frac{23}{32}C_F^2\right.$$
$$+ \left[\left(-\frac{970}{81} + \frac{224}{27}\zeta_3 + \frac{5}{9}\zeta_5 + \frac{11}{108}\pi^2\right) C_A + \left(-\frac{29}{96} + \frac{19}{6}\zeta_3 - \frac{10}{3}\zeta_5\right) C_F\right] f$$
$$\left. + \left(\frac{151}{162} - \frac{19}{27}\zeta_3 - \frac{1}{108}\pi^2\right) f^2 + \left(\frac{11}{144} - \frac{1}{6}\zeta_3\right) \frac{d^{abc}d^{abc}}{C_F d(R)} \frac{\left(\sum_f Q_f\right)^2}{\sum_f Q_f^2}\right\}.$$

$$\frac{\alpha_{g_1}(Q)}{\pi} = \frac{\alpha_{\overline{\rm MS}}(Q)}{\pi} + \left(\frac{\alpha_{\overline{\rm MS}}(Q)}{\pi}\right)^2 \left[\frac{23}{12}C_A - \frac{7}{8}C_F - \frac{1}{3}f\right]$$
$$+ \left(\frac{\alpha_{\overline{\rm MS}}(Q)}{\pi}\right)^3 \left\{\left(\frac{5437}{648} - \frac{55}{18}\zeta_5\right) C_A^2 + \left(-\frac{1241}{432} + \frac{11}{9}\zeta_3\right) C_A C_F + \frac{1}{32}C_F^2\right.$$
$$\left. + \left[\left(-\frac{3535}{1296} - \frac{1}{2}\zeta_3 + \frac{5}{9}\zeta_5\right) C_A + \left(\frac{133}{864} + \frac{5}{18}\zeta_3\right) C_F\right] f + \frac{115}{648} f^2\right\}.$$

Eliminate $\overline{\rm MS}$
Find Amazing Simplification

Lu, Kataev, Gabadadze, Sjb

Generalized Crewther Relation

$$[1 + \frac{\alpha_R(s^*)}{\pi}][1 - \frac{\alpha_{g_1}(q^2)}{\pi}] = 1$$

$$\sqrt{s^*} \simeq 0.52 Q$$

Conformal relation true to all orders in perturbation theory!

No radiative corrections to axial anomaly

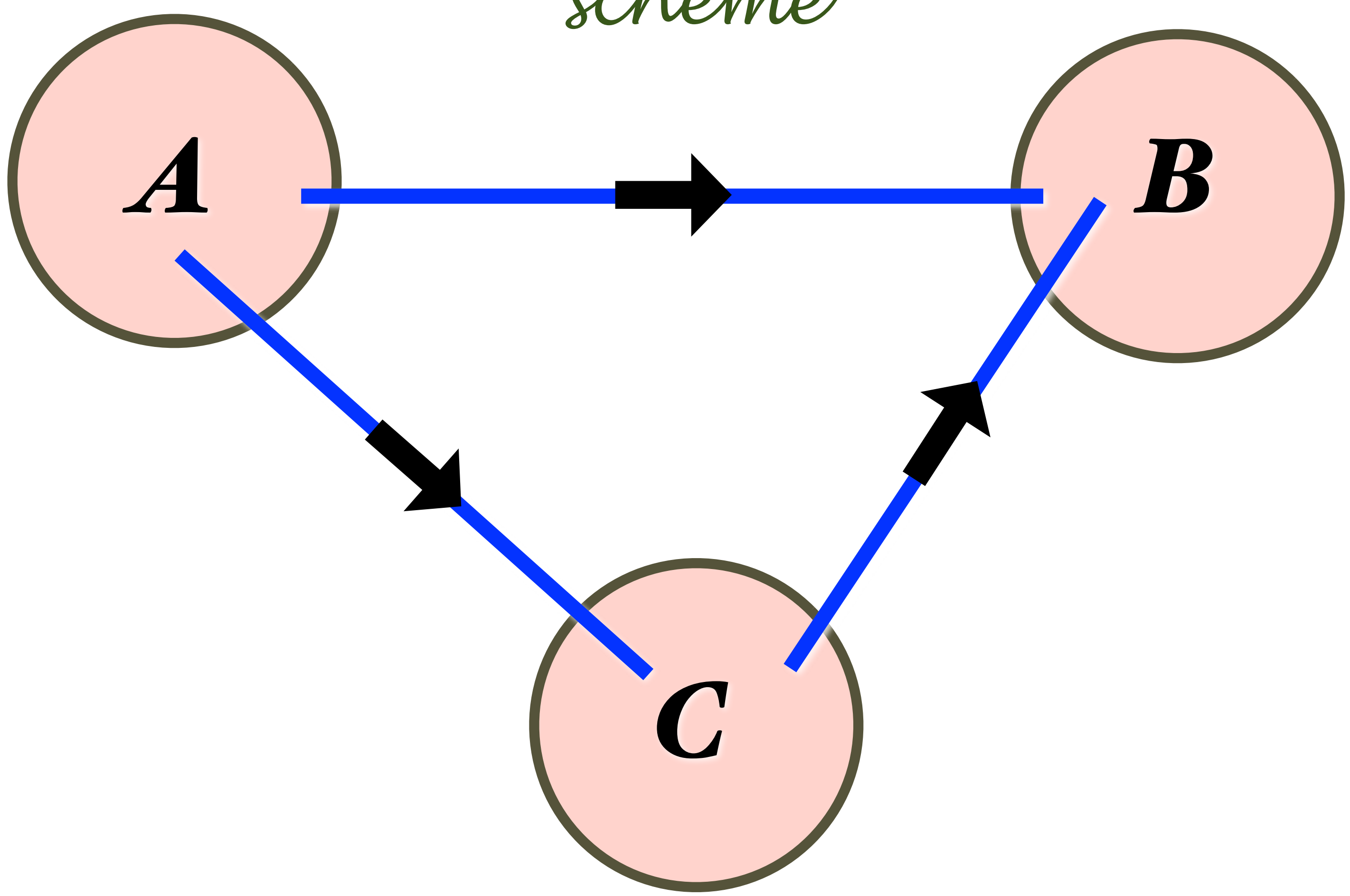
Nonconformal terms set relative scales (BLM)

No renormalization scale ambiguity!

Both observables go through new quark thresholds at commensurate scales!

Transitivity Property of Renormalization Group

Relations between observables must be independent of intermediate scheme



H. J. Lu, sjb

A → C   C → B   identical to   A → B

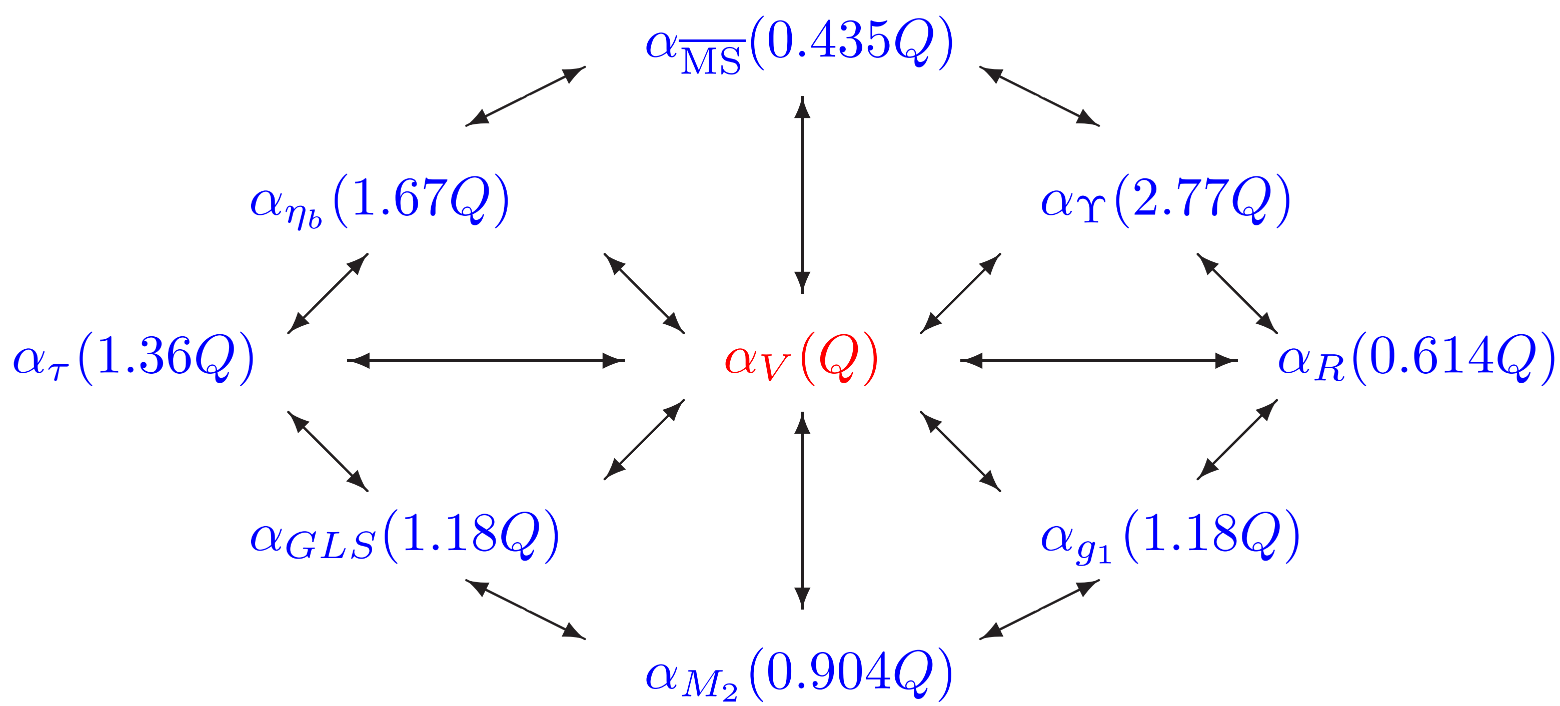
Violated by PMS!

Commensurate Scale Relations (CSR)

PMC scales in physical schemes **=>** CSR between physical observables

$$a_A(Q) = a_B(Q_1[Q]) + r_{2,0}^{AB} a_B(Q_2[Q])^2 + r_{3,0}^{AB} a_B(Q_3[Q])^3 + \cdots$$

Measuring *A* at a scale *Q* predicts value of *B* to leading order at the scale $Q_1[Q]$



Exact in special case, e.g.: $\alpha_{\tau\to\nu_\tau+\mathbf{h}}(M_\tau^2) = \alpha_{e^+e^-\to\mathbf{h}}(Q_1^2)$.

CSR: $$\ln\frac{Q_1^2}{M_\tau^2} = -\frac{19}{12} - \frac{169}{64}\frac{\alpha_{e^+e^-\to\mathbf{h}}(M_\tau^2)}{\pi} - \frac{83273}{3072}\frac{\alpha_{e^+e^-\to\mathbf{h}}(M_\tau^2)^2}{\pi^2} + \cdots$$

Highly non-trivial QCD prediction free of scheme- and scale-ambiguities!

Basic features of BLM/PMC

- It satisfies the mentioned properties: Existence, Unitary, Transitivity, Reflexivity.
- All non-conformal and scheme-dependent β-terms in perturbative series are summed into running coupling. The resultant is scheme-independent.
- Renormalons growing as ($n!\ \beta^m \alpha_s^n$) are avoided.
- The PMC method agrees with the standard QED results in the Nc-> 0 limit.

Features of BLM/PMC

- *Predictions are scheme-independent at every order*
- *Matches conformal series*
- *Commensurate Scale Relations between observables: Generalized Crewther Relation (Kataev, Lu, Rathsman, sjb)*
- *No n! Renormalon growth*
- *New scale appears at each order; n_F determined at each order - matches virtuality of quark loops*
- *Multiple Physical Scales Incorporated (Hoang, Kuhn, Tuebner, sjb)*
- *Rigorous: Satisfies all Renormalization Group Principles*
- *Realistic Estimate of Higher-Order Terms*
- *Same as Gell-Mann Low for QED* $N_C \to 0$
- *GUT: Must use the same scale setting procedure for QED, QCD*
- *Eliminates unnecessary theory error*
- *Maximal sensitivity to new physics*
- *BLM: 1039 citations. Example: BFKL intercept* (Fadin, Kim, Lipatov, Pivovarov, sjb)

Problems with traditional scale setting

- *Predictions are scheme-dependent! At every order! This fundamental flaw does not get repaired at high orders*
- *Fails to satisfy Renormalization Group Principles*
- *Guessing the scale and range is heuristic*
- *Gives wrong predictions for QED*
- *GUT: Must use the same scale-setting procedure for QED, QCD*
- *n! Renormalon growth — no convergence of pQCD*
- *Uses the same scale at each order.*
- *n_f does not reflect quark loop virtuality*
- *Multiple Physical Scales cannot be Incorporated*
- *Unrealistic Estimate of Higher-Order Terms: Only β-terms exposed by scale variation*
- *Introduces an unnecessary theory error!*
- *Distinctly different predictions for pQCD observables* See: Czakon, Fiedler, Heymes, Mitov
- *Obscures sensitivity to new physics*

Factorization Scale

- Factorization scale not the same as the renormalization scale
- Factorization scale ambiguity even for conformal theory $\beta = 0$
- Use AdS/QCD
- Factorization Scale Q_0: Boundary between nonperturbative and perturbative QCD

Deur, de Tèramond, sjb

All-Scale QCD Coupling

$$m_\rho = \sqrt{2}\kappa$$

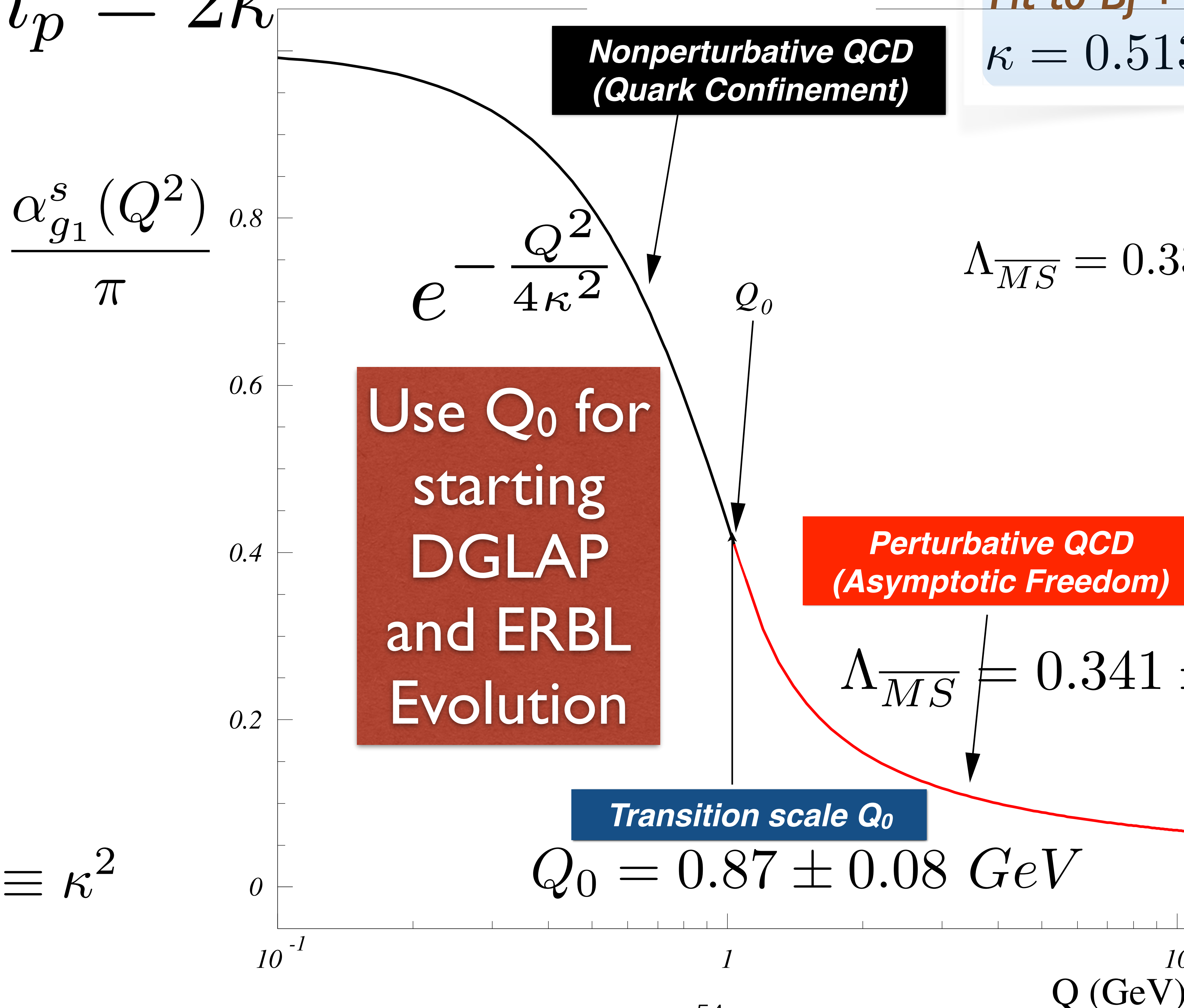
$$m_p = 2\kappa$$

Fit to Bj + DHG Sum Rules:

$$\kappa = 0.513 \pm 0.007\ GeV$$

Expt:

$$\Lambda_{\overline{MS}} = 0.339 \pm 0.016\ GeV$$



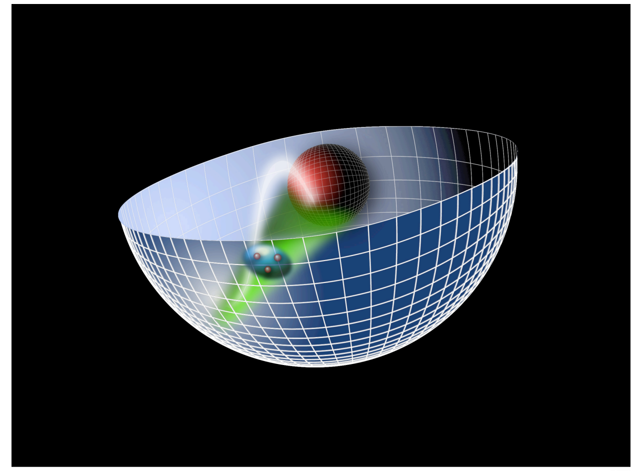
$$\lambda \equiv \kappa^2$$

$\overline{MS}$ scheme

de Tèramond, Dosch, sjb

AdS/QCD Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$



$$\zeta^2 = x(1-x)\mathrm{b}^2_\perp .$$

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique Confinement Potential!

Preserves Conformal Symmetry of the action

$$\kappa \simeq 0.5\ GeV$$

Confinement scale:

$$1/\kappa \simeq 1/3\ fm$$

- **de Alfaro, Fubini, Furlan:**
  - **Fubini, Rabinovici:**

Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

LF Holography

Baryon Equation

Superconformal Quantum Mechanics

$$\left(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2(L_B+1) + \frac{4L_B^2-1}{4\zeta^2}\right)\psi_J^+ = M^2\psi_J^+$$

$$\left(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2 L_B + \frac{4(L_B+1)^2-1}{4\zeta^2}\right)\psi_J^- = M^2\psi_J^-$$

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

S=1/2, P=+

both chiralities

Meson Equation

$$\left(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2(J-1) + \frac{4L_M^2-1}{4\zeta^2}\right)\phi_J = M^2\phi_J$$

$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Same κ!

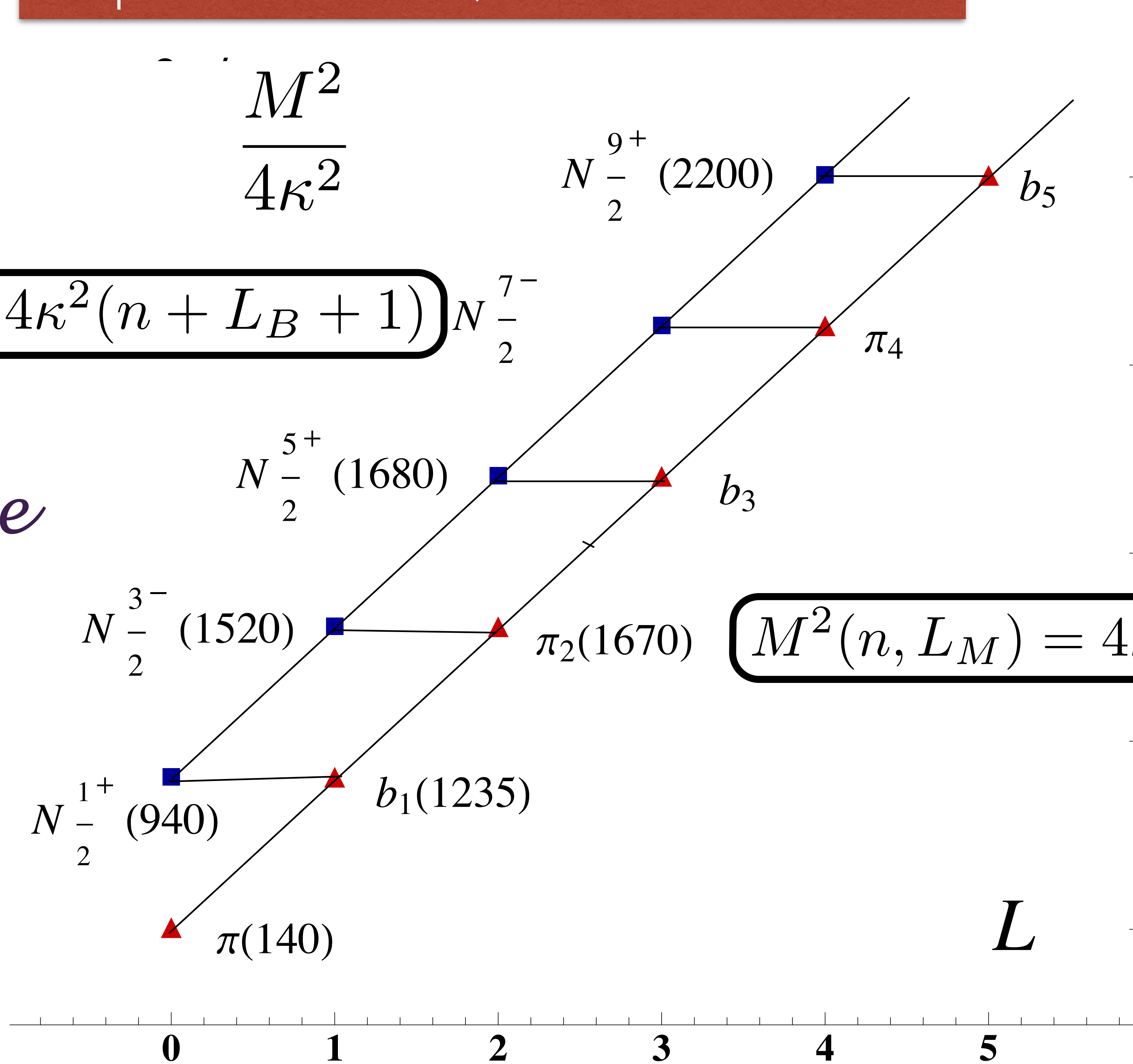
S=0, I=1 Meson is superpartner of S=1/2, I=1 Baryon

Meson-Baryon Degeneracy for $L_M = L_B + 1$

Superconformal Quantum Mechanics

$$\boxed{M^2(n, L_B) = 4\kappa^2(n + L_B + 1)}$$

Same slope

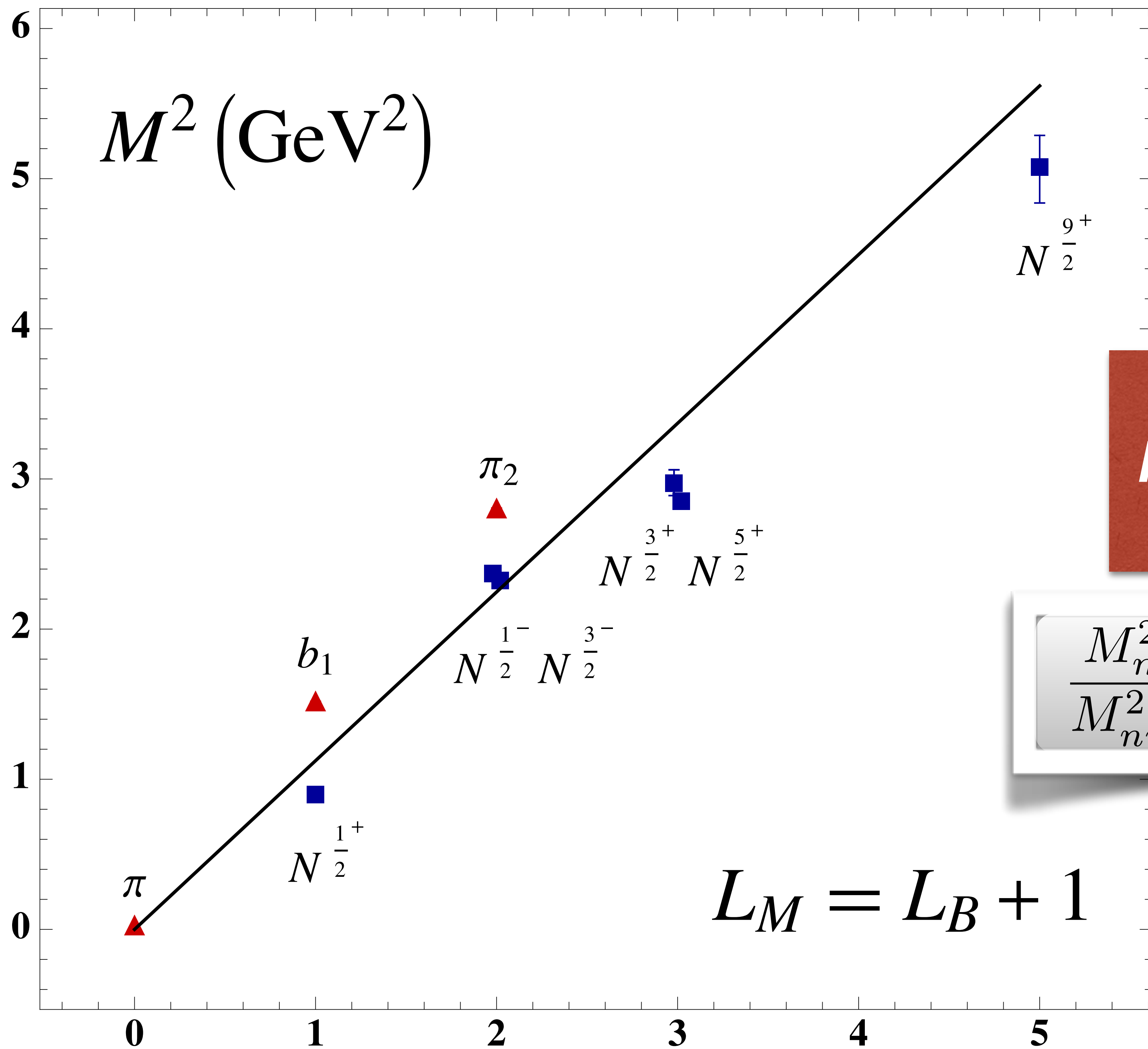


$$\boxed{M^2(n, L_M) = 4\kappa^2(n + L_M)}$$

Meson-Baryon Mass Degeneracy for $L_M=L_B+1$

$$\lambda_M^2 = \lambda_B^2 = \kappa^4$$

Superconformal AdS Light-Front Holographic QCD (LFHQCD): Identical meson and baryon spectra!



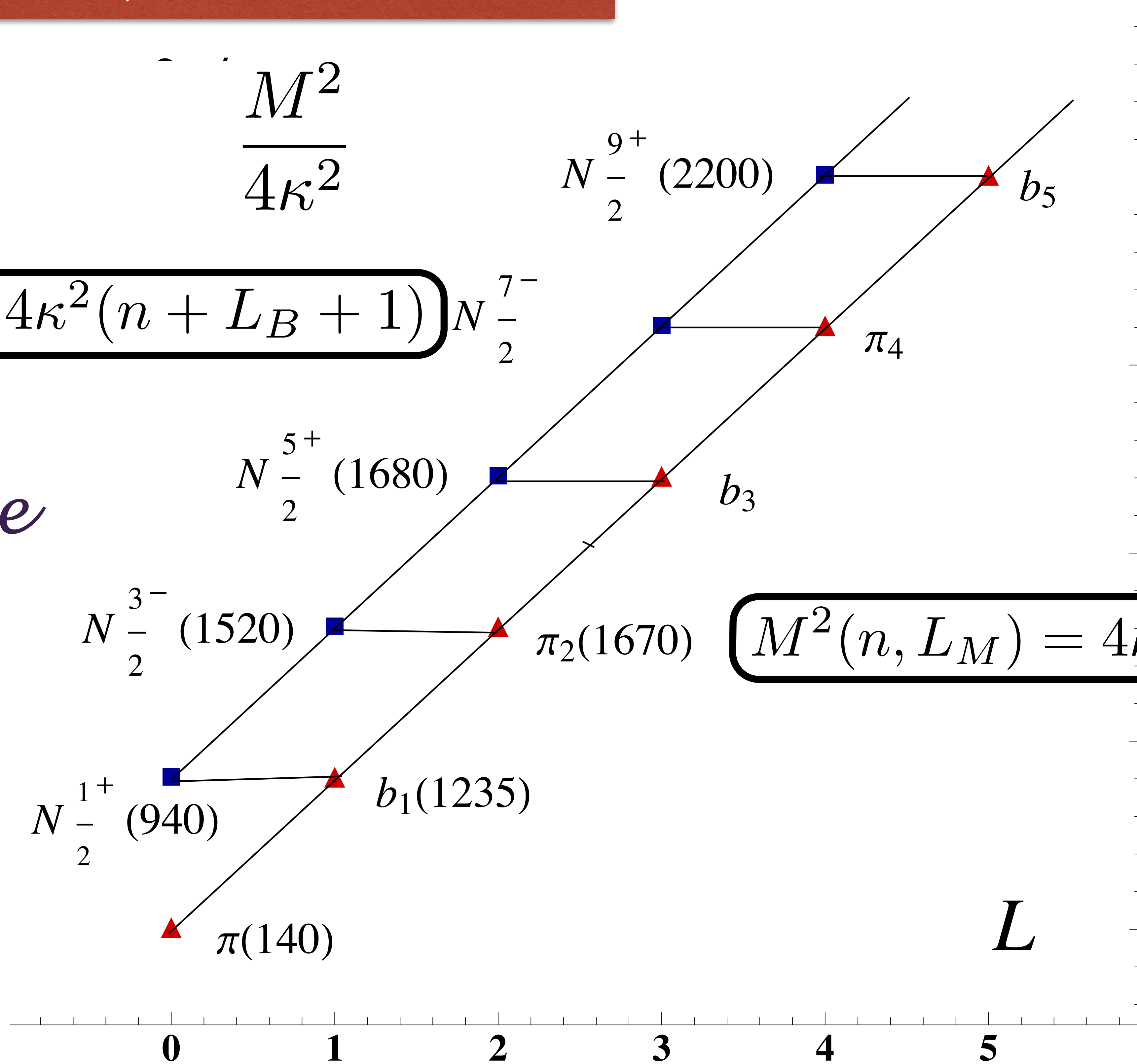
Meson-Baryon Mass Degeneracy for $L_M=L_B+1$

$$\frac{M^2_{meson}}{M^2_{nucleon}} = \frac{n+L_M}{n+L_B+1}$$

$S=0$, $I=1$ Meson is superpartner of $S=1/2$, $I=1$ Baryon

Superconformal Quantum Mechanics

de Tèramond, Dosch, sjb



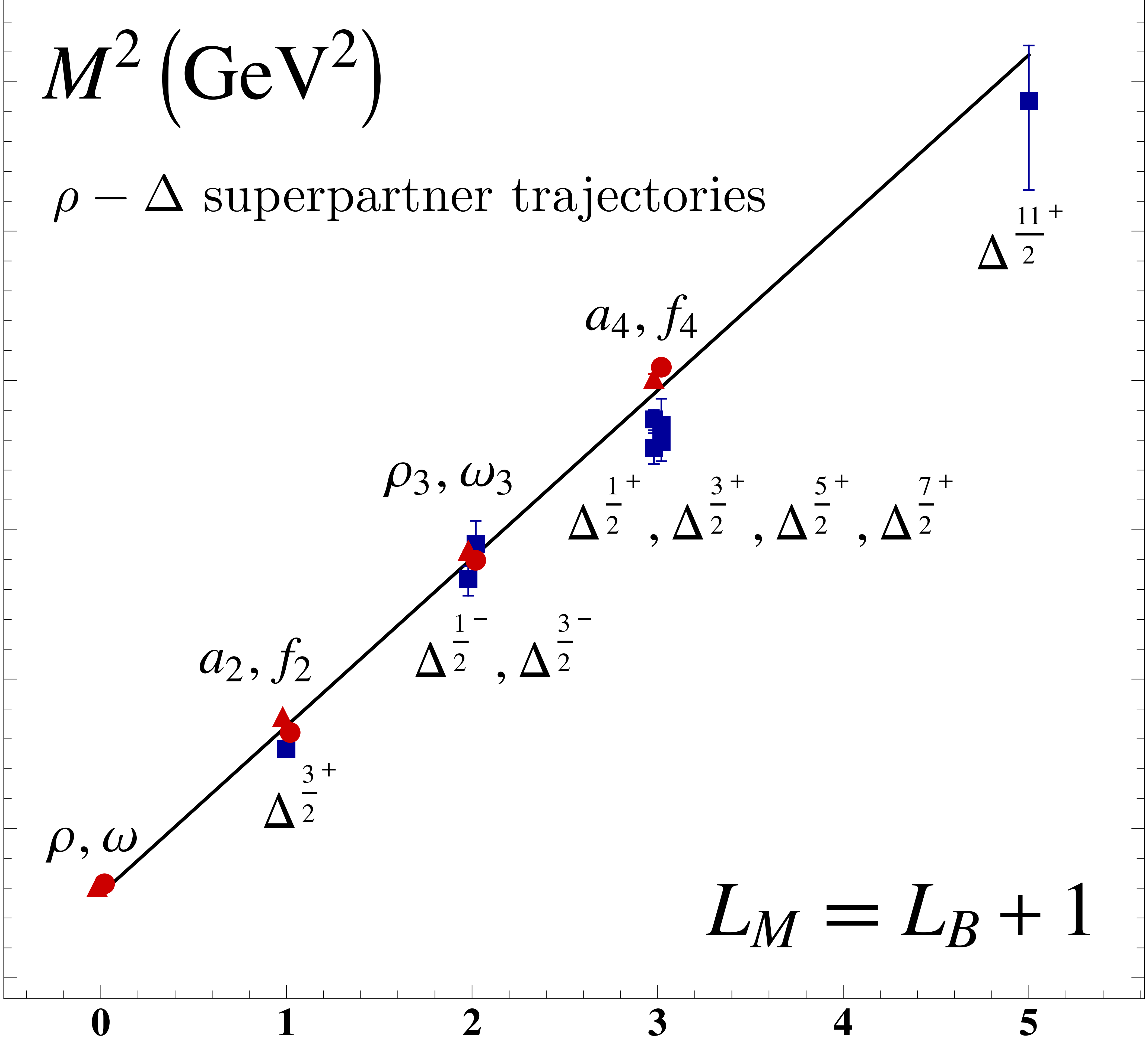
$$\boxed{M^2(n, L_B) = 4\kappa^2(n + L_B + 1)}$$

Same slope

$$\boxed{M^2(n, L_M) = 4\kappa^2(n + L_M)}$$

$$\frac{M^2_{meson}}{M^2_{nucleon}} = \frac{n + L_M}{n + L_B + 1}$$

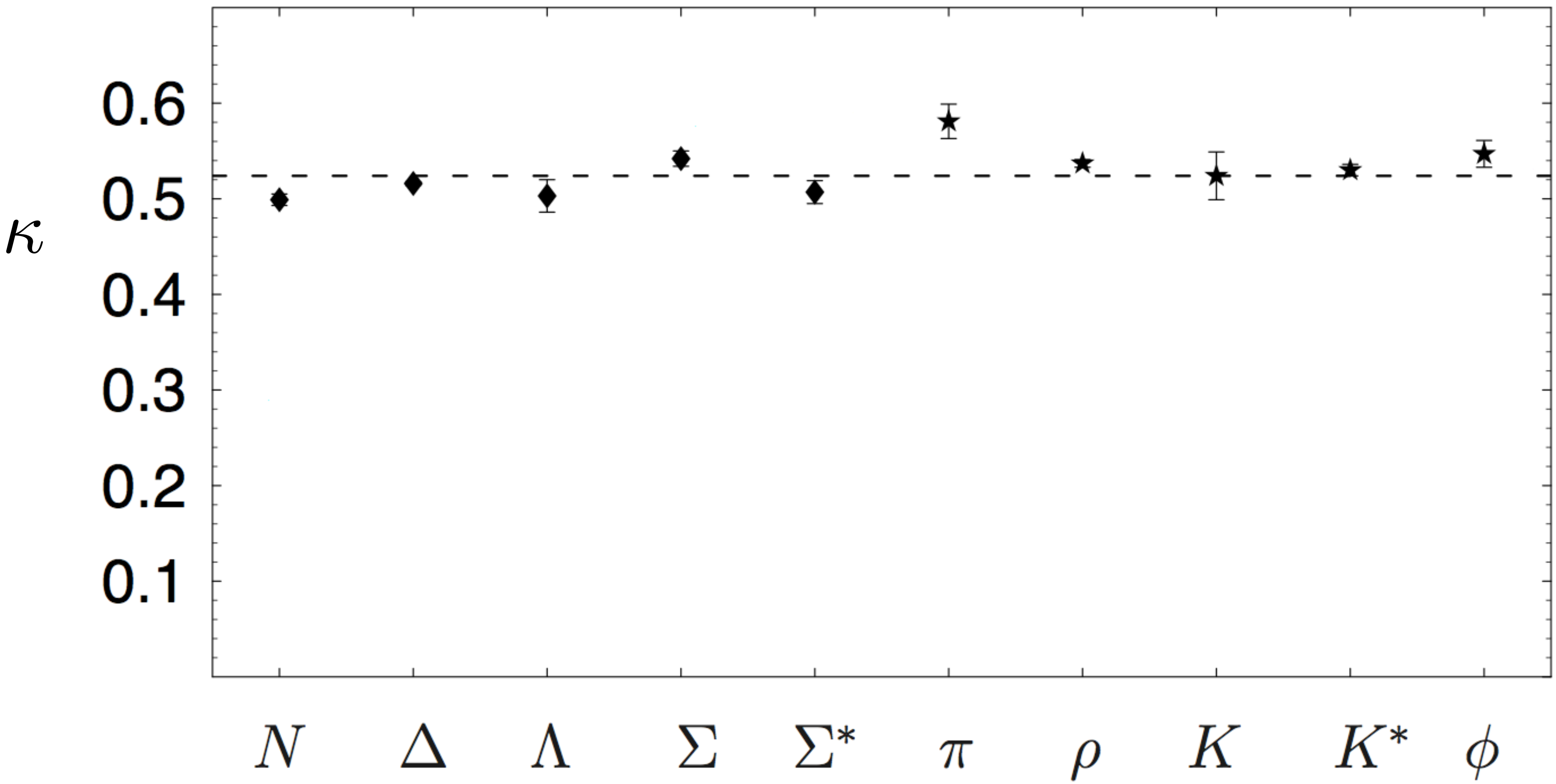
Meson-Baryon Mass Degeneracy for $L_M = L_B + 1$



Dosch, de Teramond, sjb

Dosch, de Teramond, Lorce, sjb

$$m_u = m_d = 46 \text{ MeV}, \; m_s = 357 \text{ MeV}$$



Fit to the slope of Regge trajectories, including radial excitations

Same Regge Slope for Meson, Baryons: Supersymmetric feature of hadron physics

Features of Supersymmetric Equations

- J =L+S baryon simultaneously satisfies both equations of G with L , L+1 for same mass eigenvalue

- $J^z = L^z + 1/2 = (L^z + 1) - 1/2$ $\quad S^z = \pm 1/2$

- Baryon spin carried by quark orbital angular momentum: $<J^z> = L^z + 1/2$

- Mass-degenerate meson “superpartner” with $L_M = L_B + 1$. *“Shifted meson-baryon Duality”*

Meson and baryon have same κ !

Pion is massless for $m_q = 0$

de Tèramond, Dosch, sjb

Tony Zee

"Quantum Field Theory in a Nutshell"

Dreams of Exact Solvability

"In other words, if you manage to calculate m_P it better come out proportional to Λ_{QCD} since Λ_{QCD} is the only quantity with dimension of mass around.

Light-Front Holography:

Similarly for m_ρ.

$$m_p \simeq 3.21\ \Lambda_{\overline{MS}}$$

$$m_\rho \simeq 2.2\ \Lambda_{\overline{MS}}$$

Put in precise terms, if you publish a paper with a formula giving m_ρ/m_P in terms of pure numbers such as 2 and π, the field theory community will hail you as a conquering hero who has solved QCD exactly."

$$(m_q = 0)$$

$$m_\pi = 0$$

$$\frac{m_\rho}{m_P} = \frac{1}{\sqrt{2}}$$

$$\frac{\Lambda_{\overline{MS}}}{m_\rho} = 0.455 \pm 0.031$$

Bjorken sum rule defines effective charge $\alpha_{g1}(Q^2)$

$$\int_0^1 dx[g_1^{ep}(x,Q^2) - g_1^{en}(x,Q^2)] \equiv \frac{g_a}{6}[1 - \frac{\alpha_{g1}(Q^2)}{\pi}]$$

- ***Can be used as standard QCD coupling***
- ***Well measured***
- ***Asymptotic freedom at large Q^2***
- ***Computable at large Q^2 in any pQCD scheme***
- ***Universal β_0, β_1***

Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

- Consider five-dim gauge fields propagating in AdS_5 space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$S = -\frac{1}{4}\int d^4x\, dz\, \sqrt{g}\, e^{\varphi(z)}\, \frac{1}{g_5^2}\, G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \to g_{YM}(\zeta)$

- Coupling measured at momentum scale Q

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta\, J_0(\zeta Q)\, \alpha_s^{AdS}(\zeta)$$

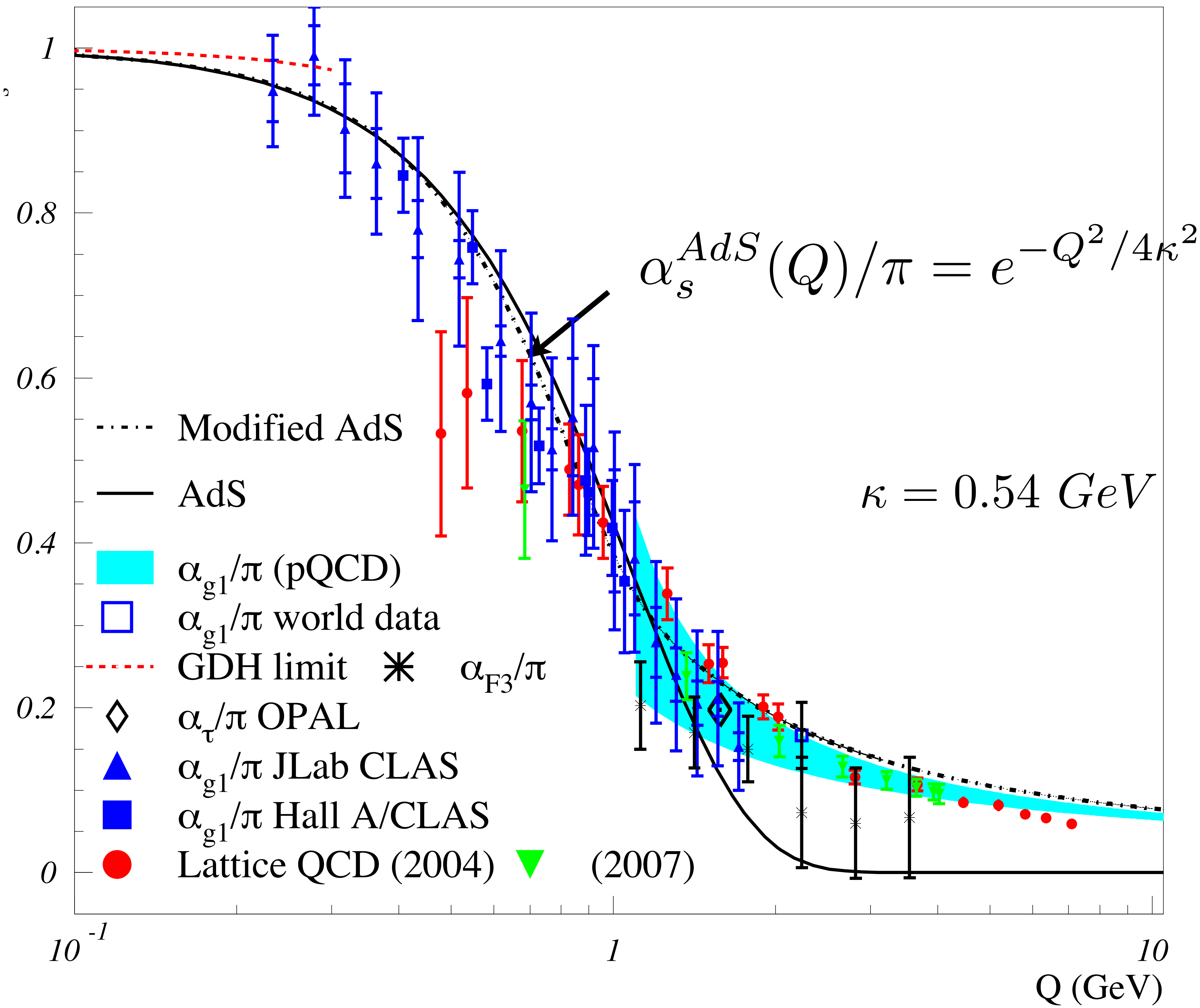
- Solution

$$\boxed{\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0)\, e^{-Q^2/4\kappa^2}.}$$

where the coupling α_s^{AdS} incorporates the non-conformal dynamics of confinement

Analytic, defined at all scales, IR Fixed Point

$$\frac{\alpha_s(Q)}{\pi}$$



AdS/QCD dilaton captures the higher twist corrections to effective charges for Q < 1 GeV

$$e^{\varphi} = e^{+\kappa^2 z^2}$$

Deur, de Teramond, sjb

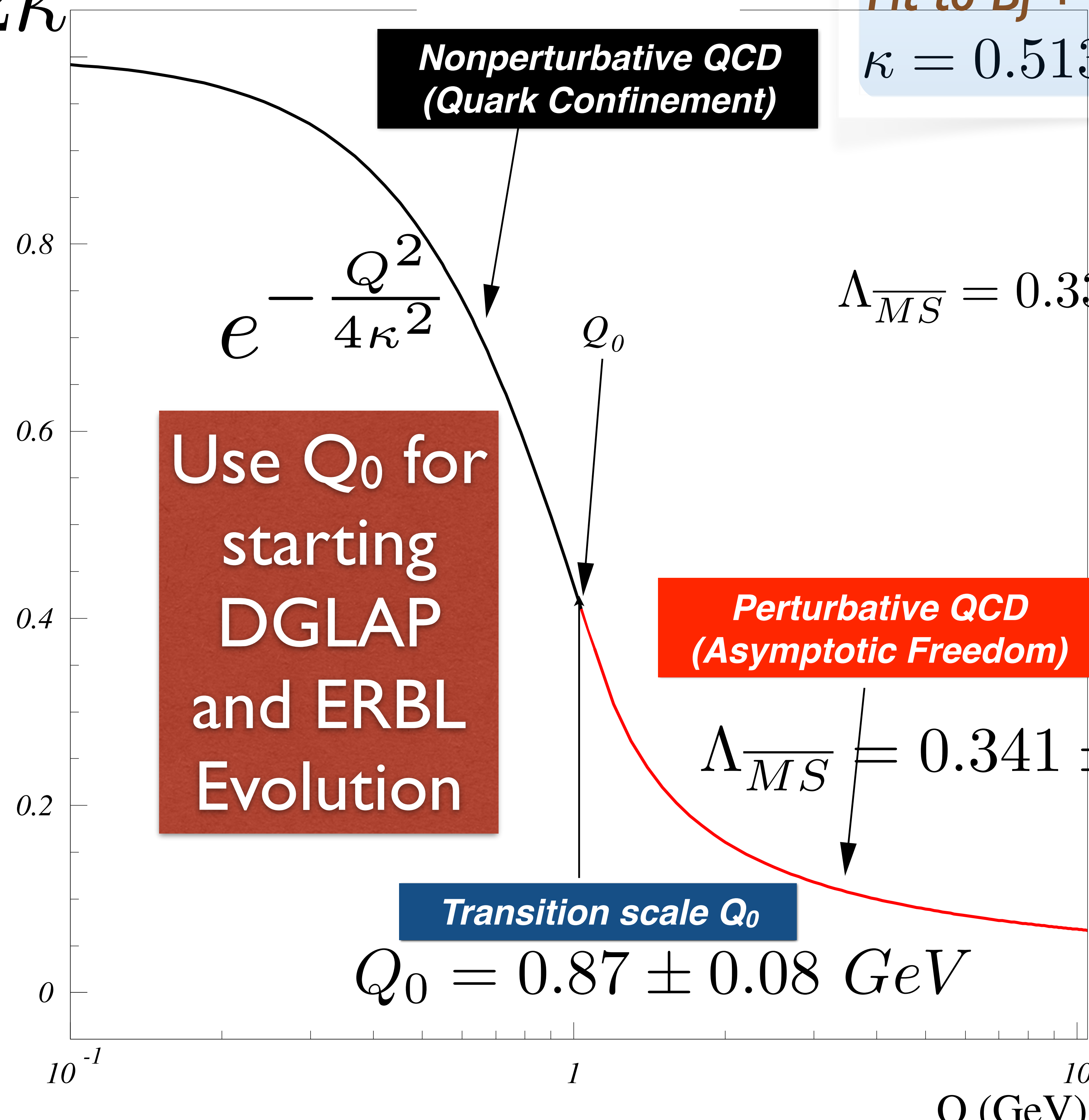
Deur, de Tèramond, sjb

All-Scale QCD Coupling

$$m_\rho = \sqrt{2}\kappa$$

$$m_p = 2\kappa$$

$$\frac{\alpha^s_{g_1}(Q^2)}{\pi}$$



Fit to Bj + DHG Sum Rules:

$$\kappa = 0.513 \pm 0.007\ GeV$$

Expt:

$$\Lambda_{\overline{MS}} = 0.339 \pm 0.016\ GeV$$

$\overline{MS}$ scheme

$$\lambda \equiv \kappa^2$$

de Tèramond, Dosch, Lorce, sjb

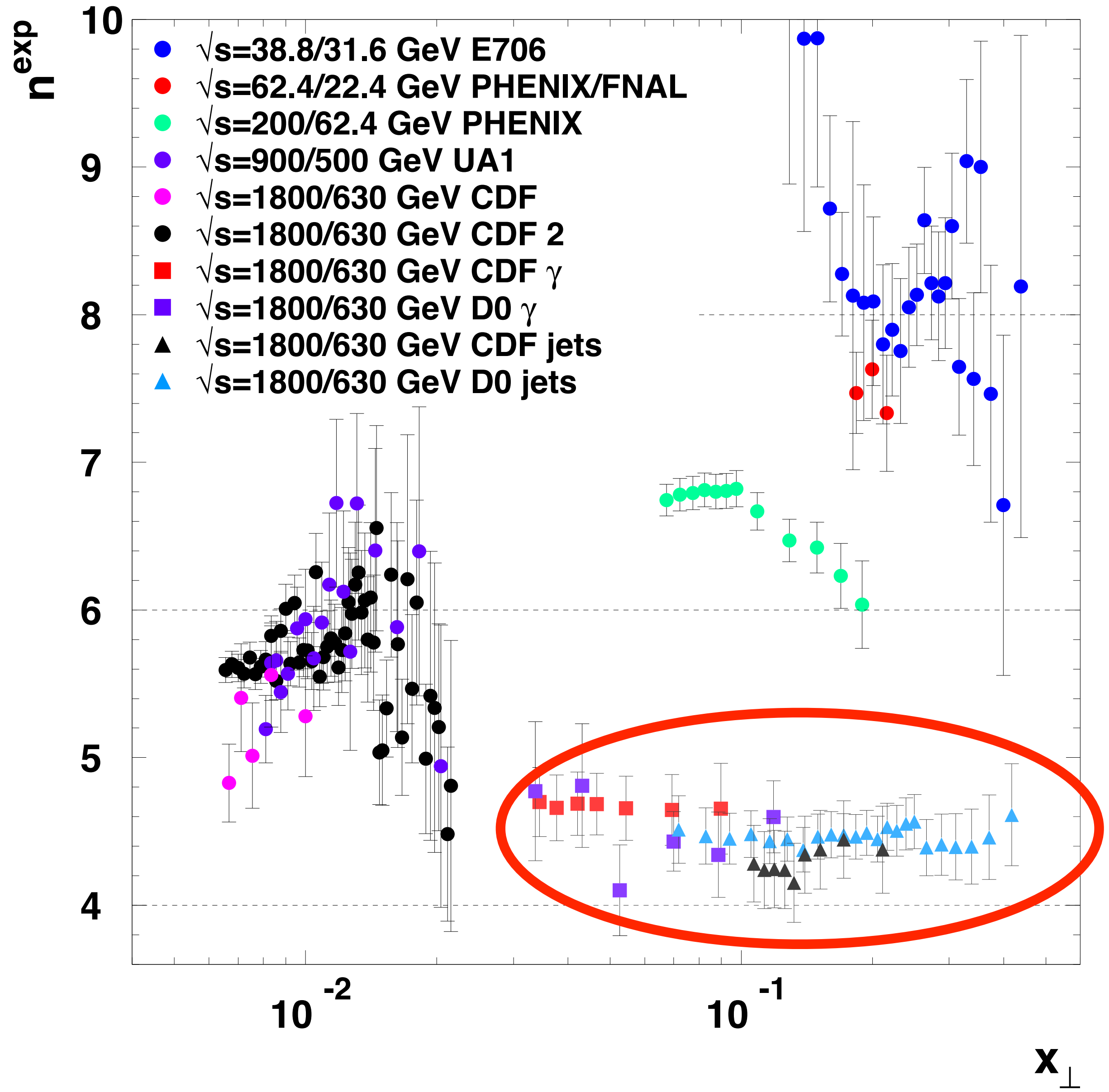
Future Directions for AdS/QCD

- **Hadronization at the Amplitude Level**
- **Diffractive dissociation of pion and proton to jets**
- **Factorization Scale for ERBL, DGLAP evolution: Q_o**
- **Calculate Sivers Effect including FSI and ISI**
- **Compute Tetraquark Spectroscopy: Sequential Clusters**
- **Update SU(6) spin-flavor symmetry**
- **Heavy Quark States: Supersymmetry, not conformal**
- **Compute higher Fock states; e.g. Intrinsic Heavy Quarks**
- **Nuclear States — Hidden Color**
- **Basis LF Quantization** *Vary, sjb*

QCD Myths

- Anti-Shadowing is Universal
- ISI and FSI are higher twist effects and universal
- High transverse momentum hadrons arise only from jet fragmentation -- baryon anomaly!
- heavy quarks only from gluon splitting
- renormalization scale cannot be fixed
- QCD condensates are vacuum effects
- Infrared Slavery
- Nuclei are composites of nucleons only
- Real part of DVCS arbitrary

$$E\frac{d\sigma}{d^3p}(pp \to HX) = \frac{F(x_T, \theta_{cm} = \pi/2)}{p_T^n}$$

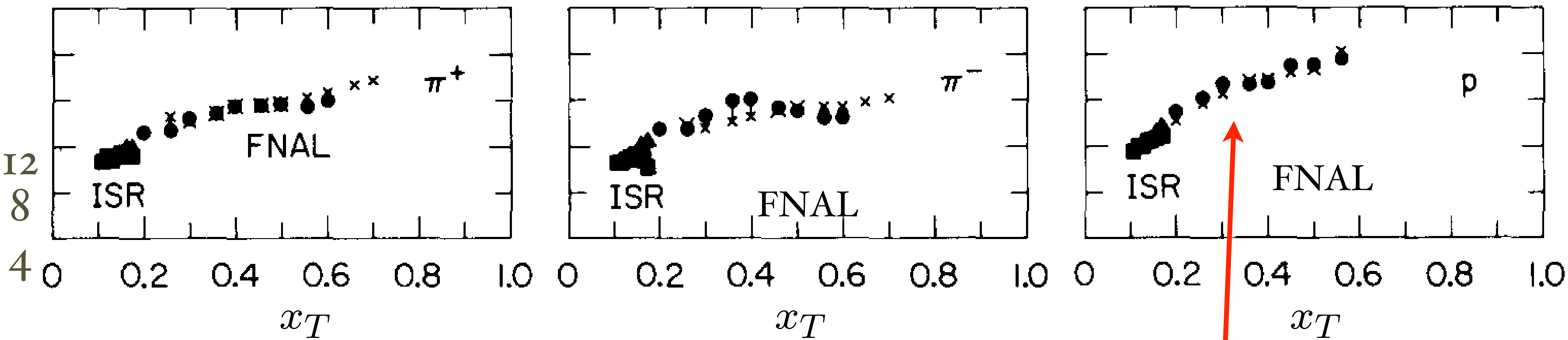


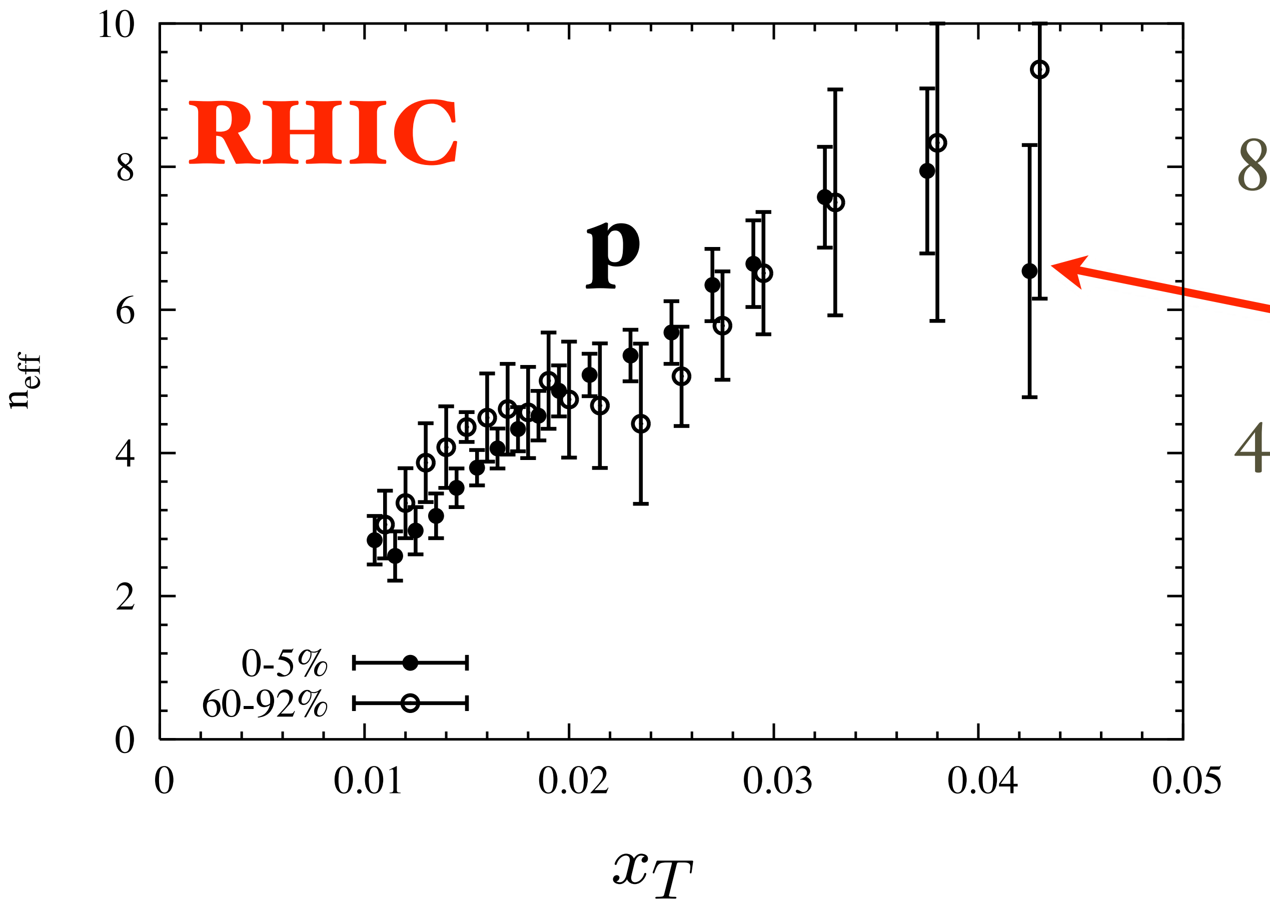
Photons and Jets agree with PQCD x_T scaling Hadrons do not!

Arleo,Hwang, Sickles, sjb

- Significant increase of the hadron $n^{\rm exp}$ with $x_\perp$
  - $n^{\rm exp} \simeq 8$ at large $x_\perp$
- Huge contrast with photons and jets !
  - $n^{\rm exp}$ constant and slight above 4 at all $x_\perp$

$$E\frac{d\sigma}{d^3p}(pp \to HX) = \frac{F(x_T,\theta_{CM})}{p_T^{\,n_{eff}}}$$



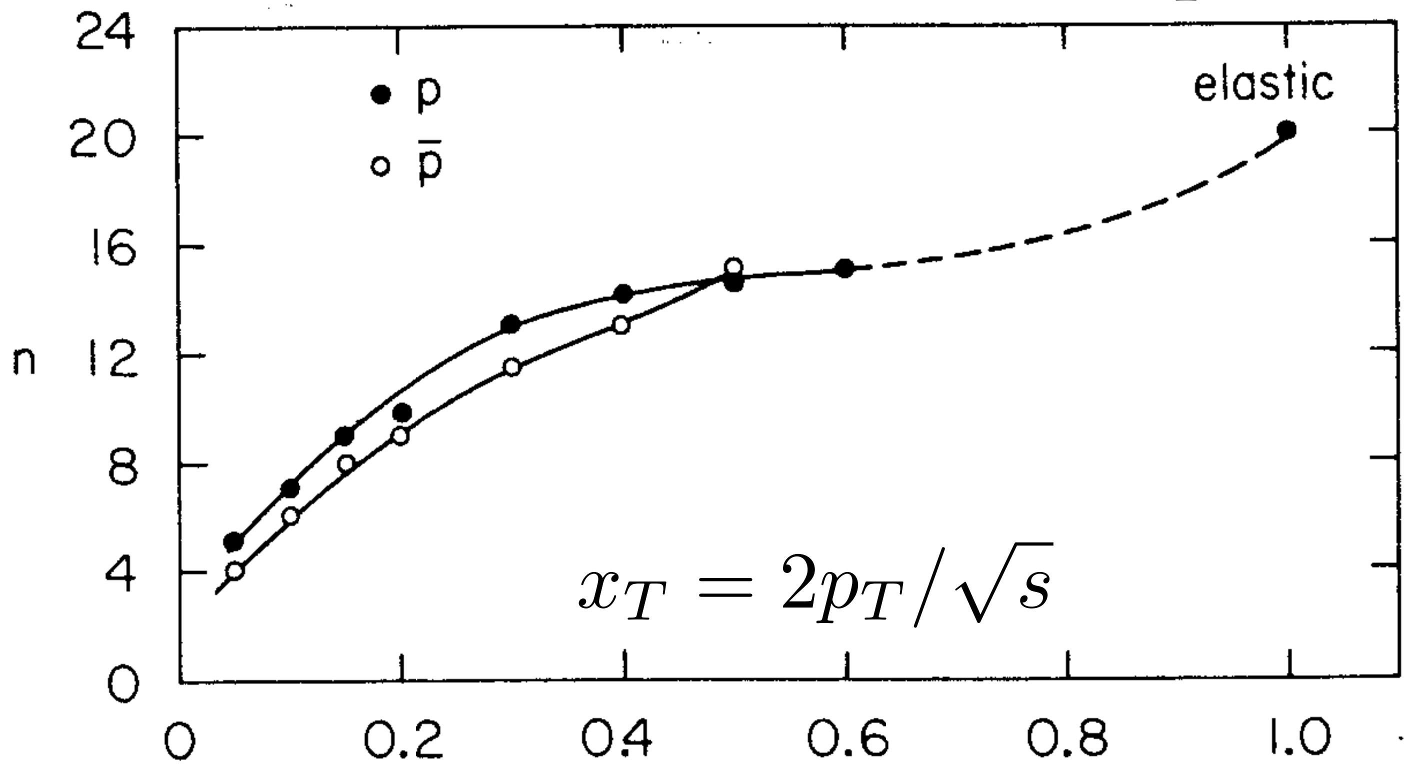


$$E\frac{d\sigma}{d^3p}(pp \to pX) = \frac{F(x_T,\theta_{CM})}{p_T^{12}}$$

$$E\frac{d\sigma}{d^3p}(pp \to pX) = \frac{F(x_T,\theta_{CM})}{p_T^{8}}$$

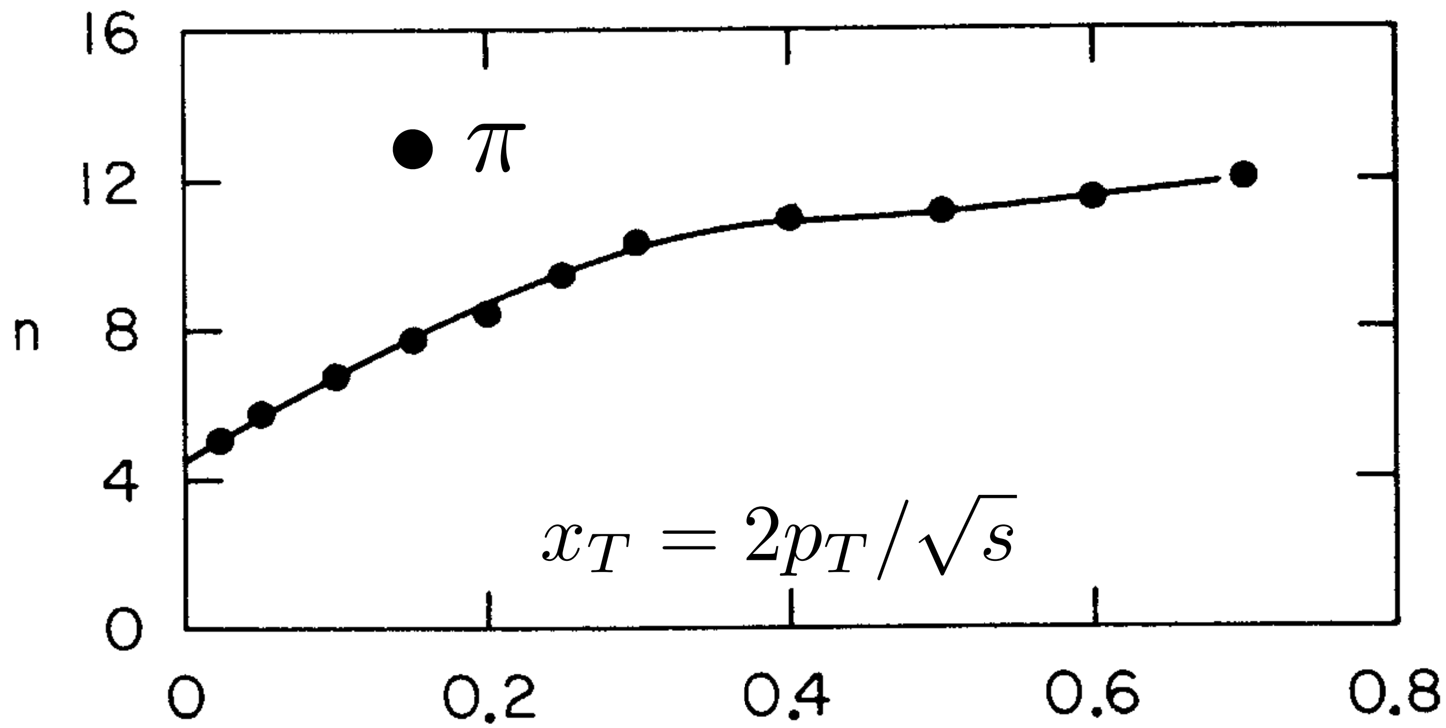
Trend consistent with RHIC at small x_T

$$E\frac{d\sigma}{d^3p}(pp \to HX) = \frac{F(x_T, \theta_{cm} = \pi/2)}{p_T^n}$$



Clear evidence for higher-twist contributions

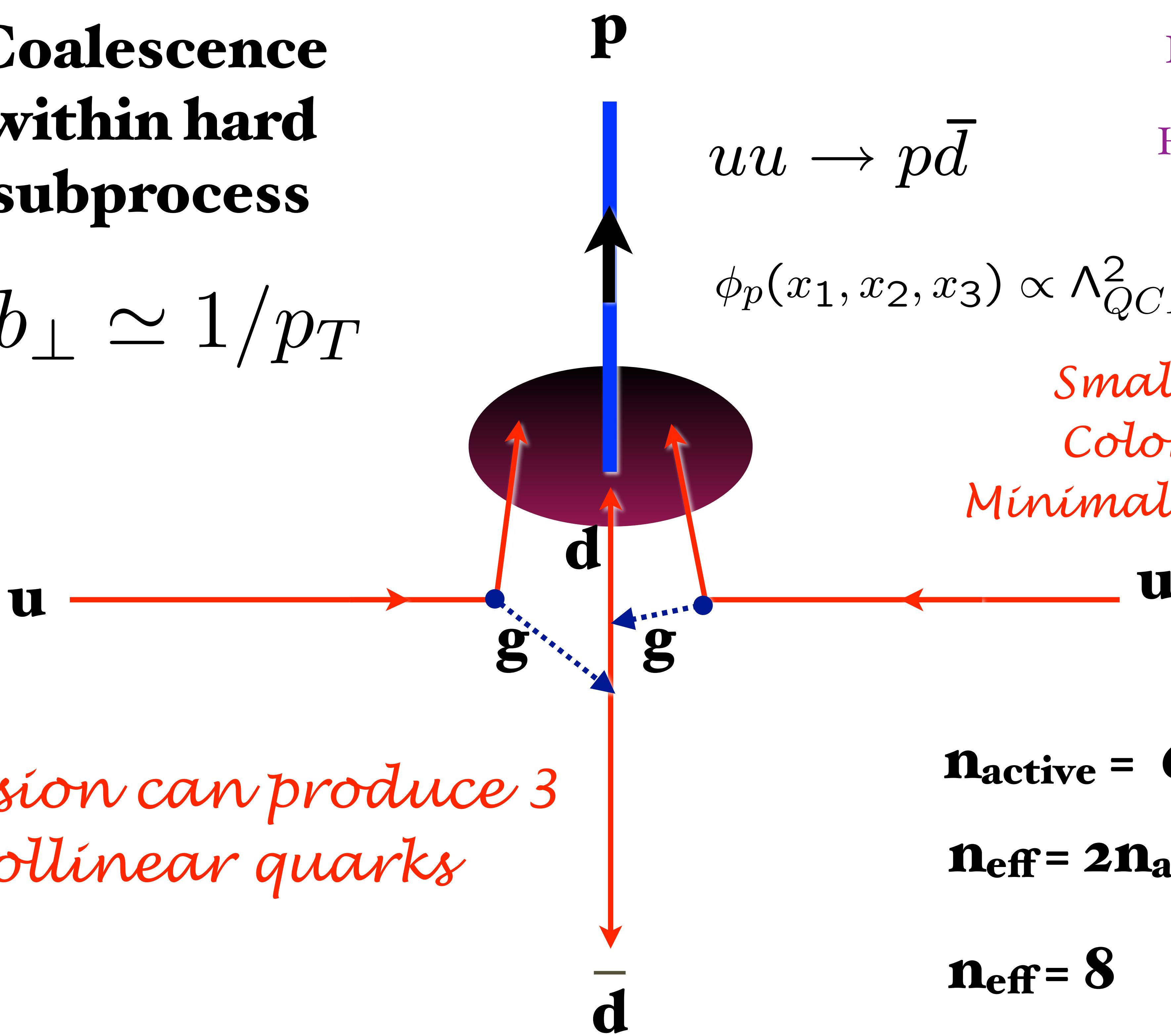
J. W. Cronin, SSI 1974



Baryon can be made directly within hard subprocess

Coalescence within hard subprocess

$$b_\perp \simeq 1/p_T$$



$$uu \longrightarrow p\bar{d}$$

$$\phi_p(x_1, x_2, x_3) \propto \Lambda^2_{QCD}$$

Bjorken
Blankenbecler, Gunion, sjb
Berger, sjb
Hoyer, et al: Semi-Exclusive

Small color-singlet
Color Transparent
Minimal same-side energy

Collision can produce 3 collinear quarks

$\mathbf{n_{active} = 6}$

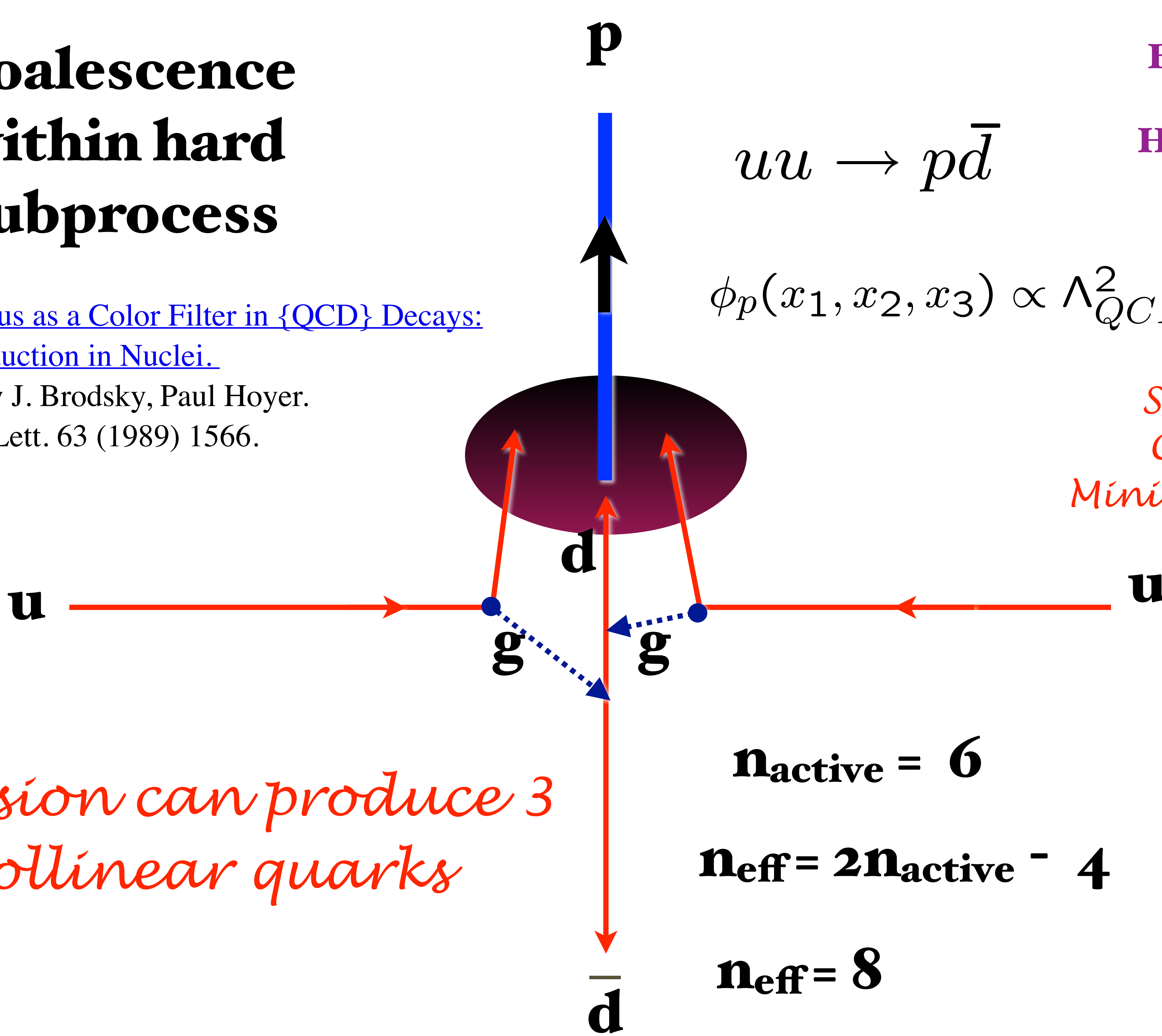
$\mathbf{n_{eff} = 2n_{active} - 4}$

$\mathbf{n_{eff} = 8}$

Baryon can be made directly within hard subprocess

Coalescence within hard subprocess

[The Nucleus as a Color Filter in {QCD} Decays: Hadroproduction in Nuclei.]()
By Stanley J. Brodsky, Paul Hoyer.
Phys.Rev.Lett. 63 (1989) 1566.



$$uu \to p\bar{d}$$

$$\phi_p(x_1, x_2, x_3) \propto \Lambda^2_{QCD}$$

Bjorken
Blankenbecler, Gunion, sjb
Berger, sjb
Hoyer, et al: Semi-Exclusive

Sickles; sjb

Small color-singlet
Color Transparent
Minimal same-side energy

Explains Baryon anomaly

$$qq \to B\bar{q}$$

Collision can produce 3 collinear quarks

$n_{active} = 6$

$n_{eff} = 2n_{active} - 4$

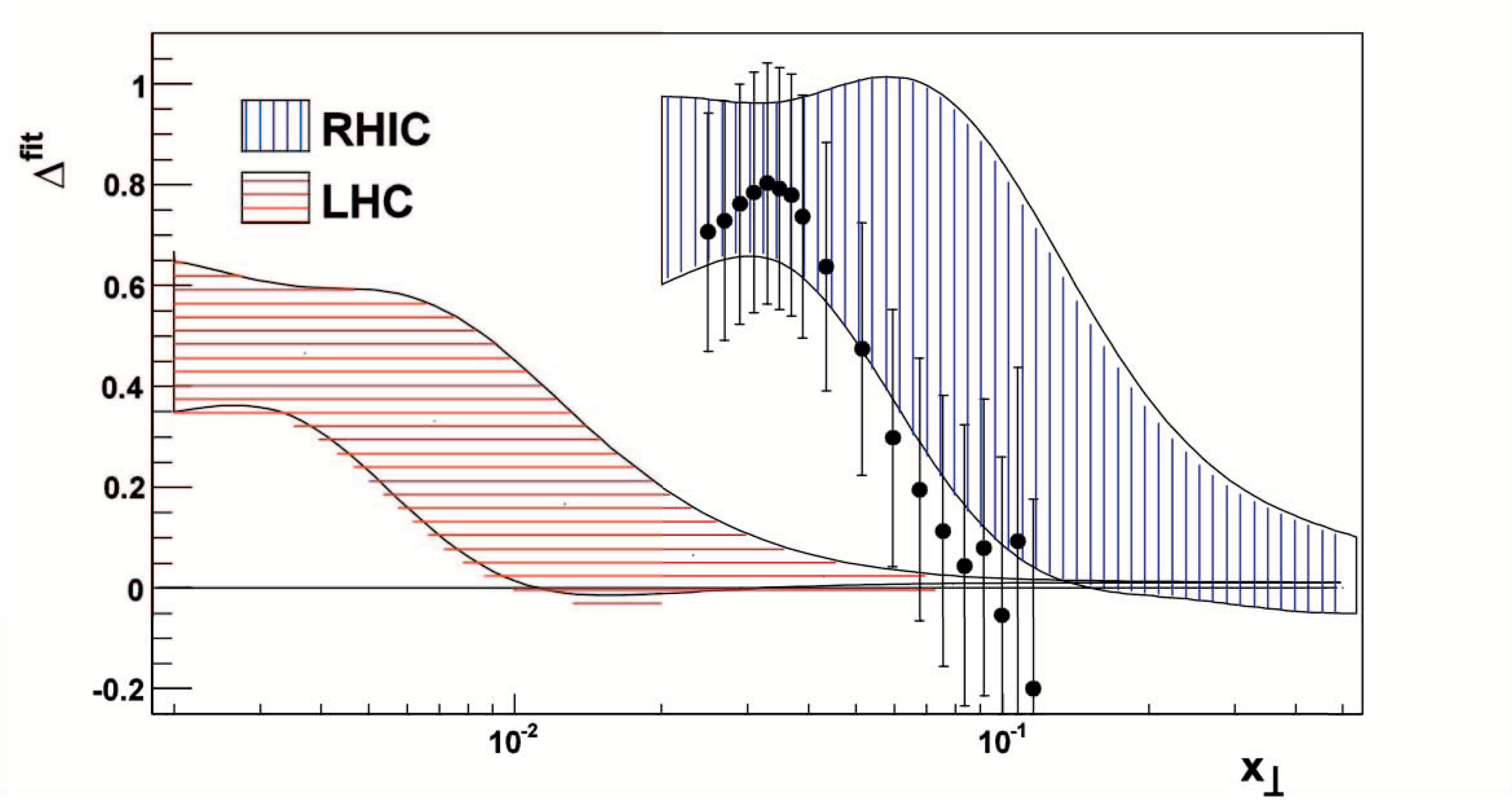
$n_{eff} = 8$

RHIC/LHC predictions

PHENIX results

Scaling exponents from $\sqrt{s} = 500$ GeV preliminary data

[A. Bezilevsky, APS Meeting]

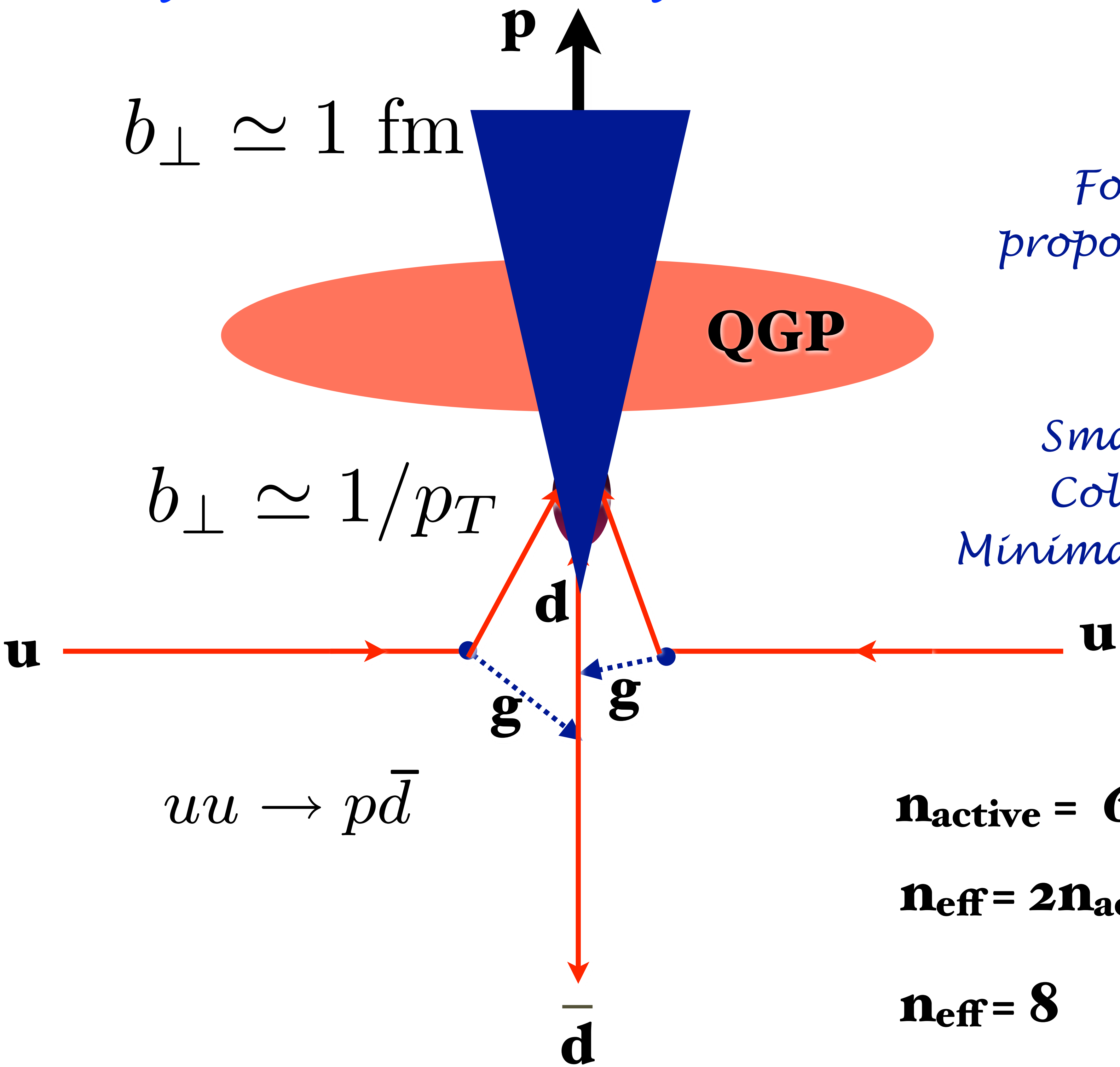


- Magnitude of Δ and its $x_\perp$-dependence consistent with predictions

$$\Delta = n_{expt} - n_{PQCD}$$

Arleo, Hwang, Sickles, sjb

Baryon made directly within hard subprocess



Formation Time proportional to Energy

Small color-singlet
Color Transparent
Minimal same-side energy

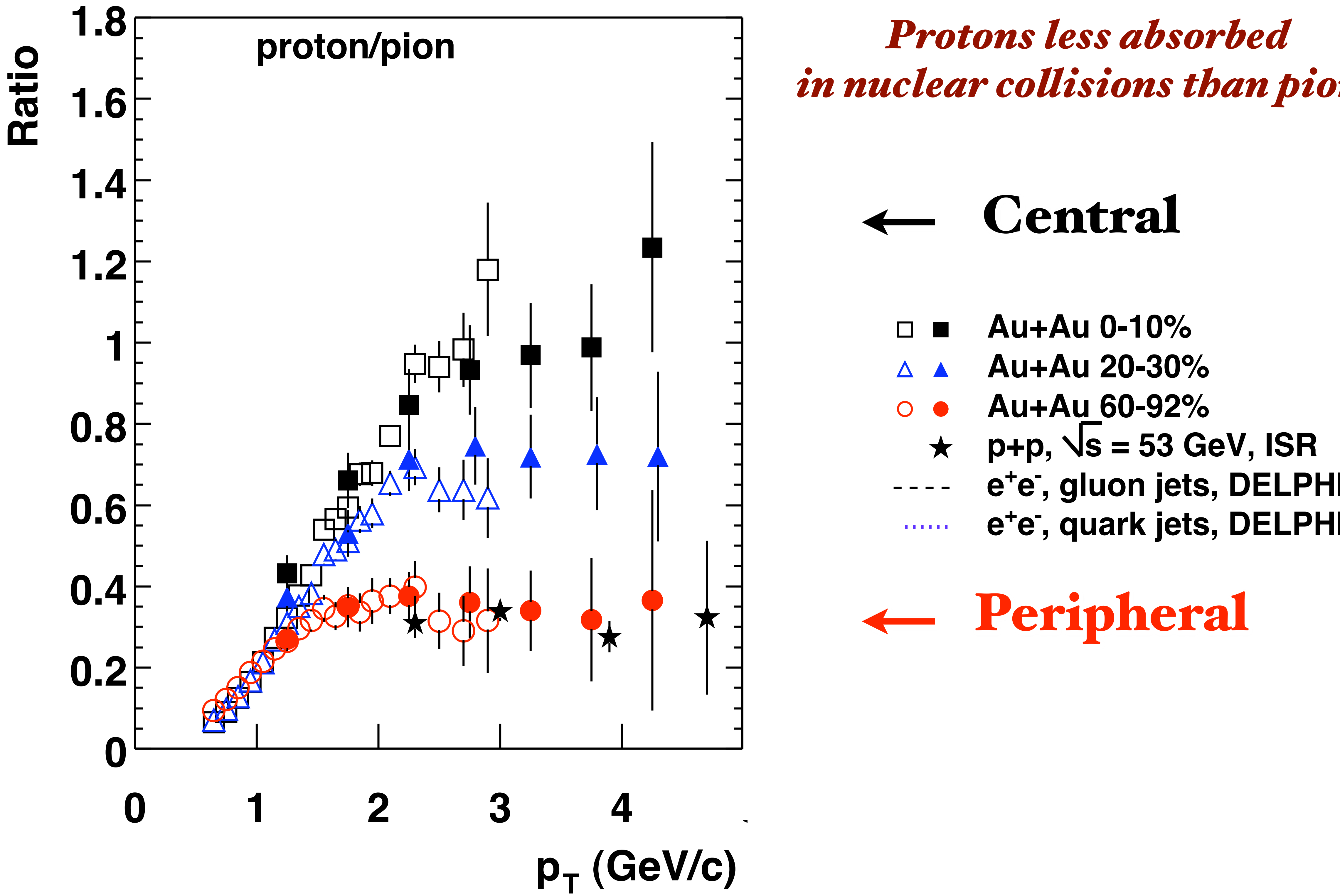
$\mathbf{n_{active}} = 6$

$\mathbf{n_{eff}} = 2\mathbf{n_{active}} - 4$

$\mathbf{n_{eff}} = 8$

S. S. Adler *et al*. PHENIX Collaboration *Phys. Rev. Lett*. **91**, 172301 (2003).

Particle ratio changes with centrality!

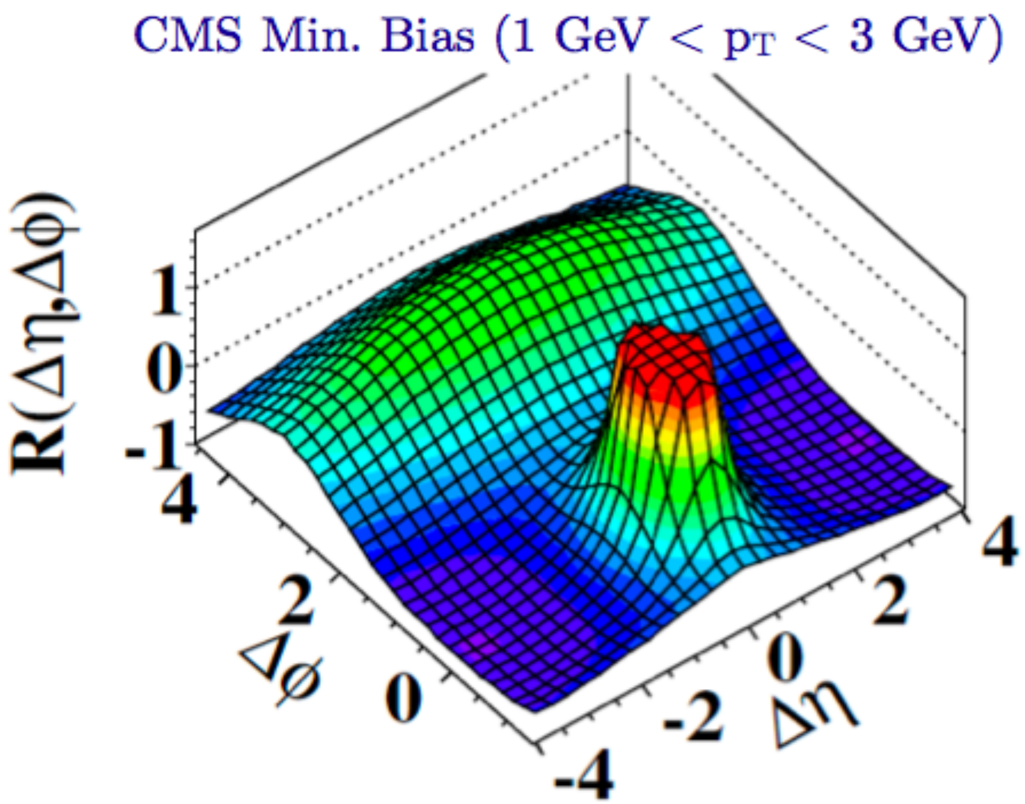


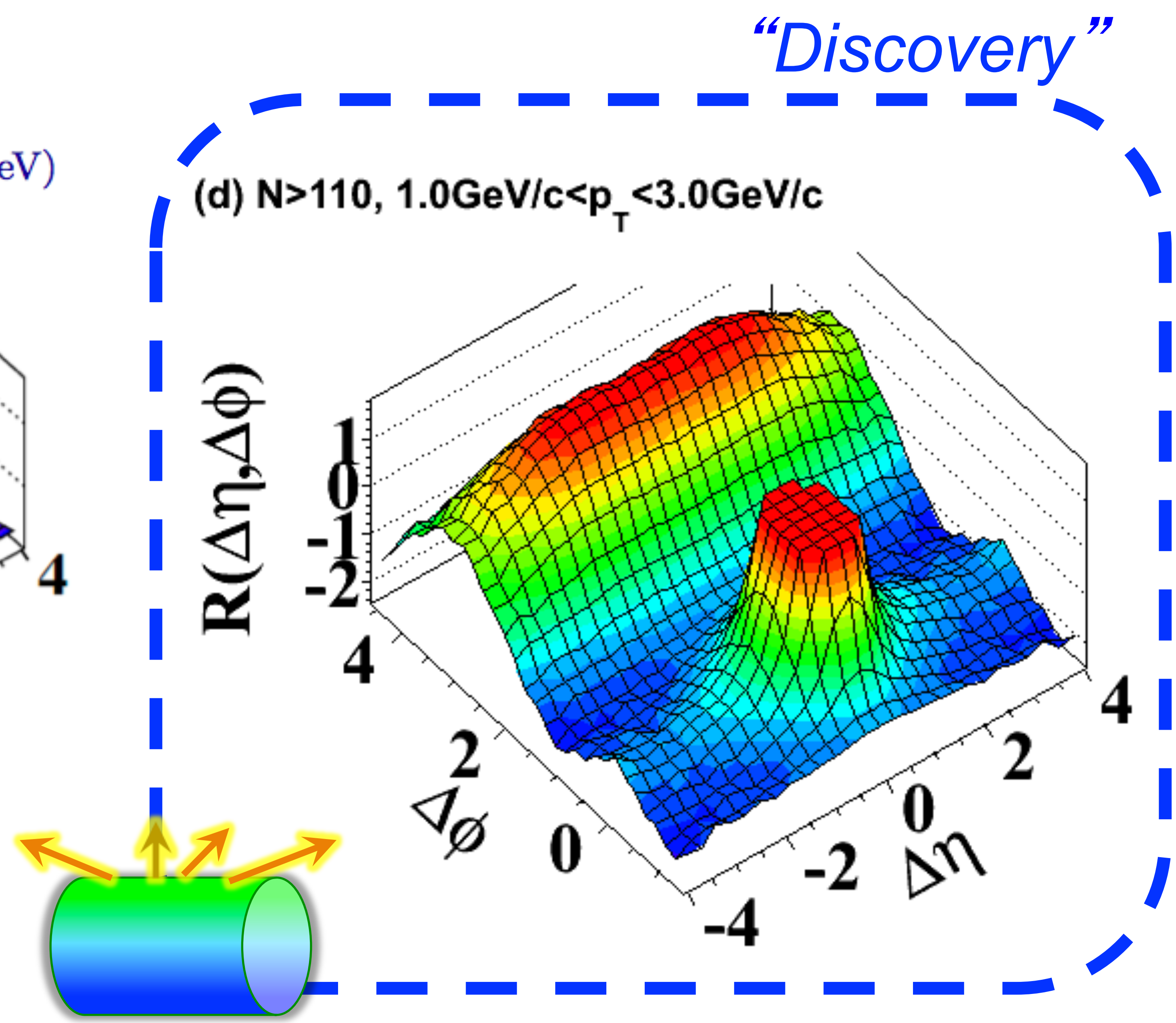
Protons less absorbed in nuclear collisions than pions

Central

Peripheral

Two particle correlations: CMS results

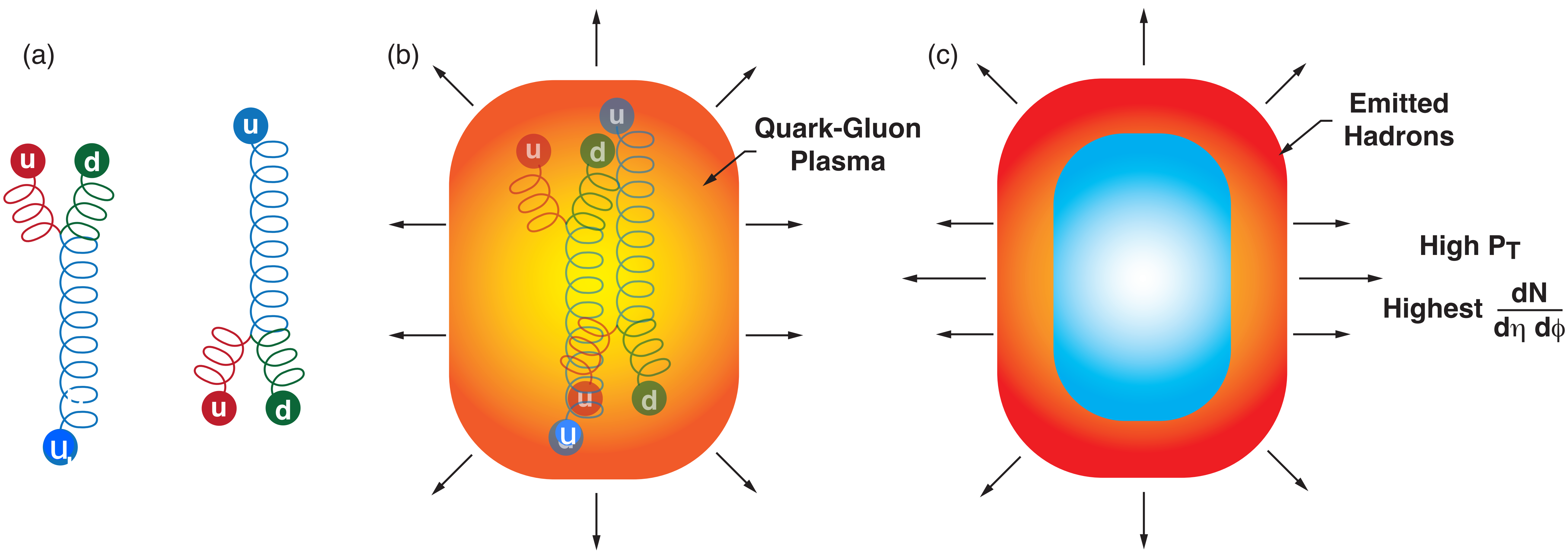




- Ridge: Distinct long range correlation in η collimated around ΔΦ≈ 0 for two hadrons in the intermediate 1 < p_T, q_T < 3 GeV

Possible origin of same-side CMS ridge in p p Collisions

Bjorken, Goldhaber, sjb



$$\vec{V} = \sum_{i=1}^{N} [\cos 2\phi_i \hat{x} + \sin 2\phi_i \hat{y}]$$

Possible multiparticle ridge-like correlations in very high multiplicity proton-proton collisions

Bjorken, Goldhaber, sjb

We suggest that this "ridge"-like correlation may be a reflection of the rare events generated by the collision of aligned flux tubes connecting the valence quarks in the wave functions of the colliding protons.

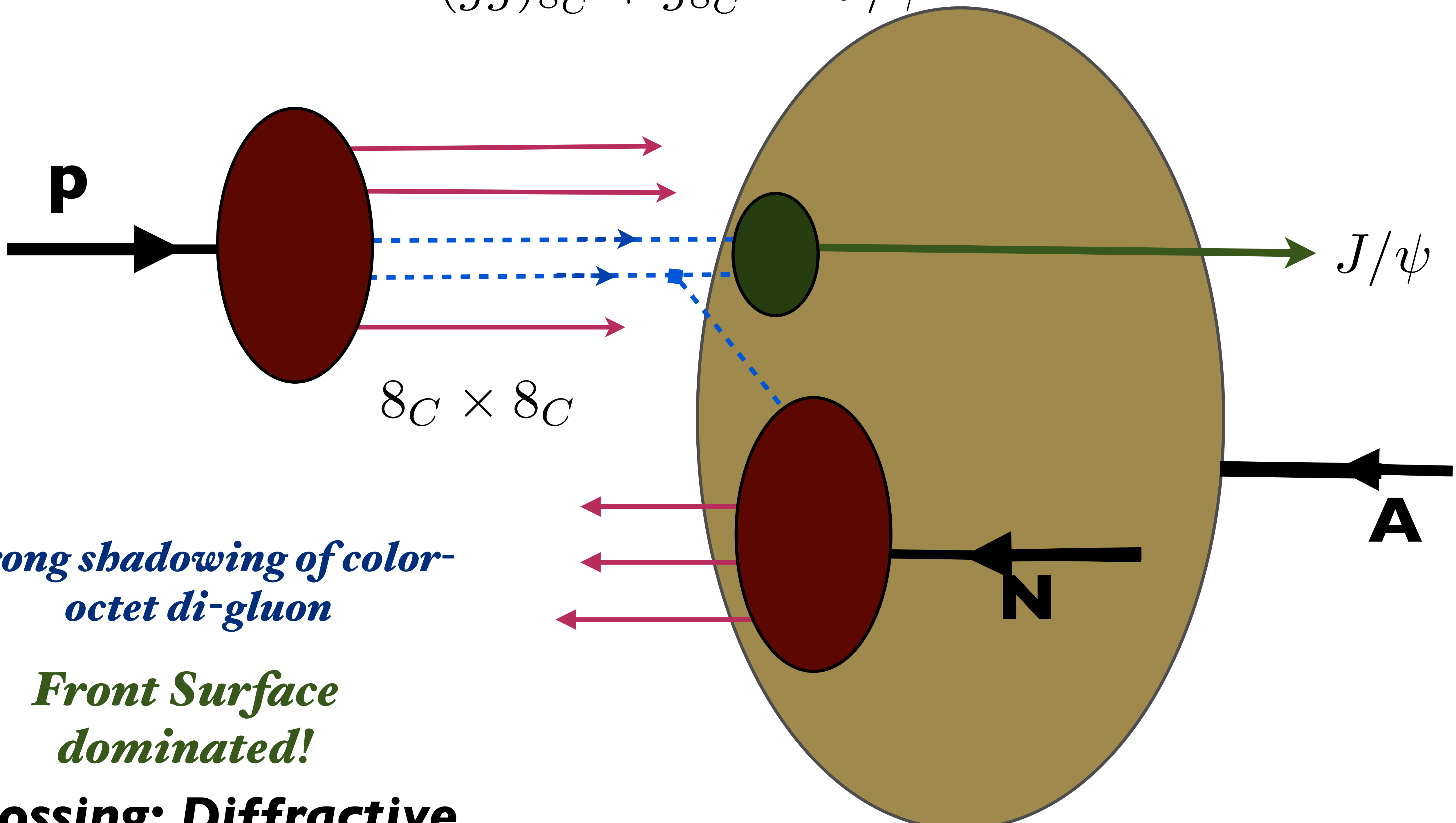
The "spray" of particles resulting from the approximate line source produced in such inelastic collisions then gives rise to events with a strong correlation between particles produced over a large range of both positive and negative rapidity.

Forward rapidity y ~4

$$pA \rightarrow J/\psi X$$

Zhu, sjb

$$(gg)_{8_C} + g_{8_C} \rightarrow J/\psi$$



Strong shadowing of color-octet di-gluon

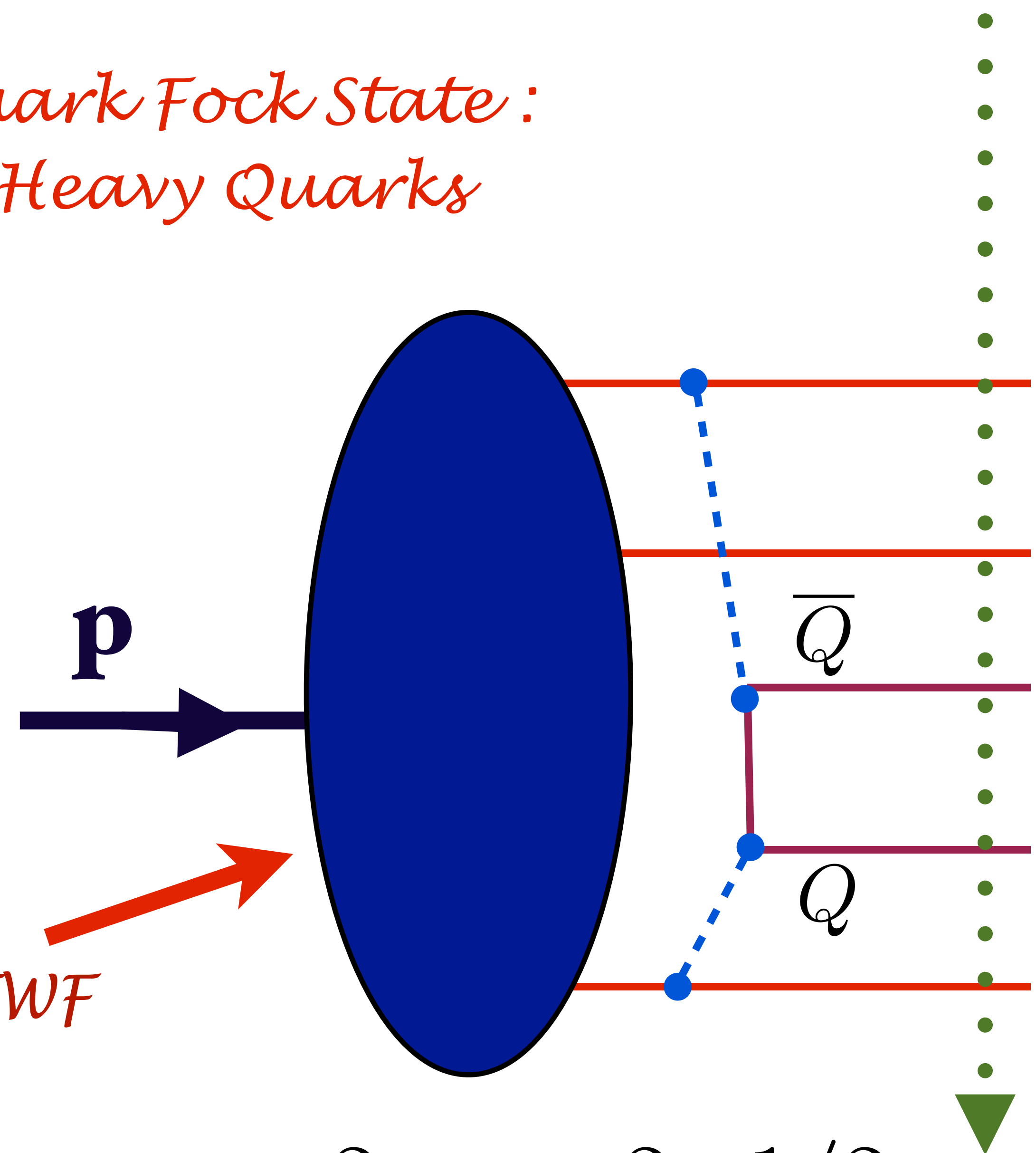
Front Surface dominated!

Crossing: Diffractive & pomeron exchange

Double-gluon subprocess

Proton 5-quark Fock State :
Intrinsic Heavy Quarks

Fixed LF time



Use AdS/QCD LFWF

QCD predicts Intrinsic Heavy Quarks at high x!

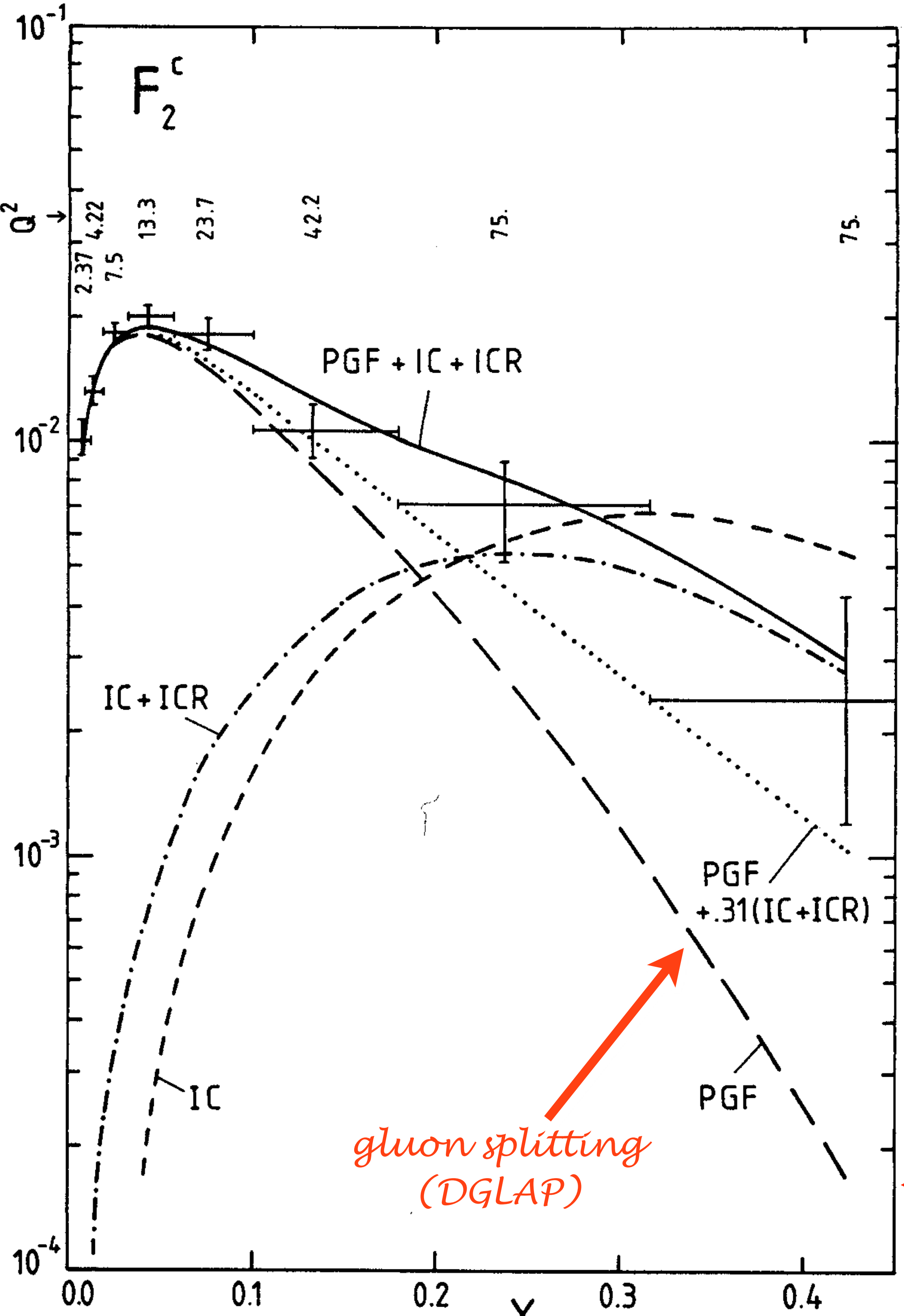
Minimal off-shellness

$$x_Q \propto (m_Q^2 + k_\perp^2)^{1/2}$$

Probability (QED) $\propto \frac{1}{M_\ell^4}$

Probability (QCD) $\propto \frac{1}{M_Q^2}$

Collins, Ellis, Gunion, Mueller, sjb
Polyakov, et al.

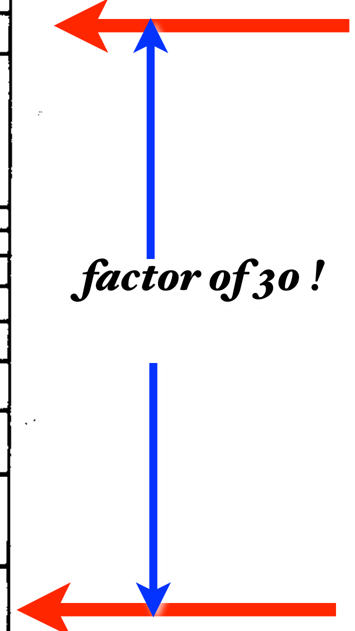


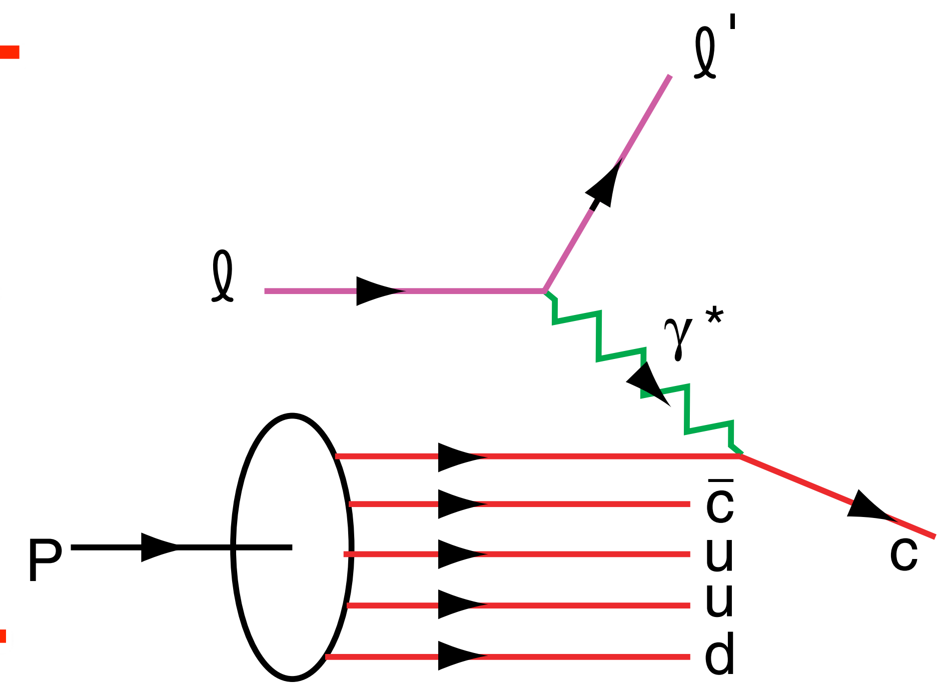
Measurement of Charm Structure Function!

J. J. Aubert et al. [European Muon Collaboration], "Production Of Charmed Particles In 250-Gev Mu+ - Iron Interactions," Nucl. Phys. B 213, 31 (1983).

First Evidence for Intrinsic Charm

Hoyer, Peterson, Sakai, sjb





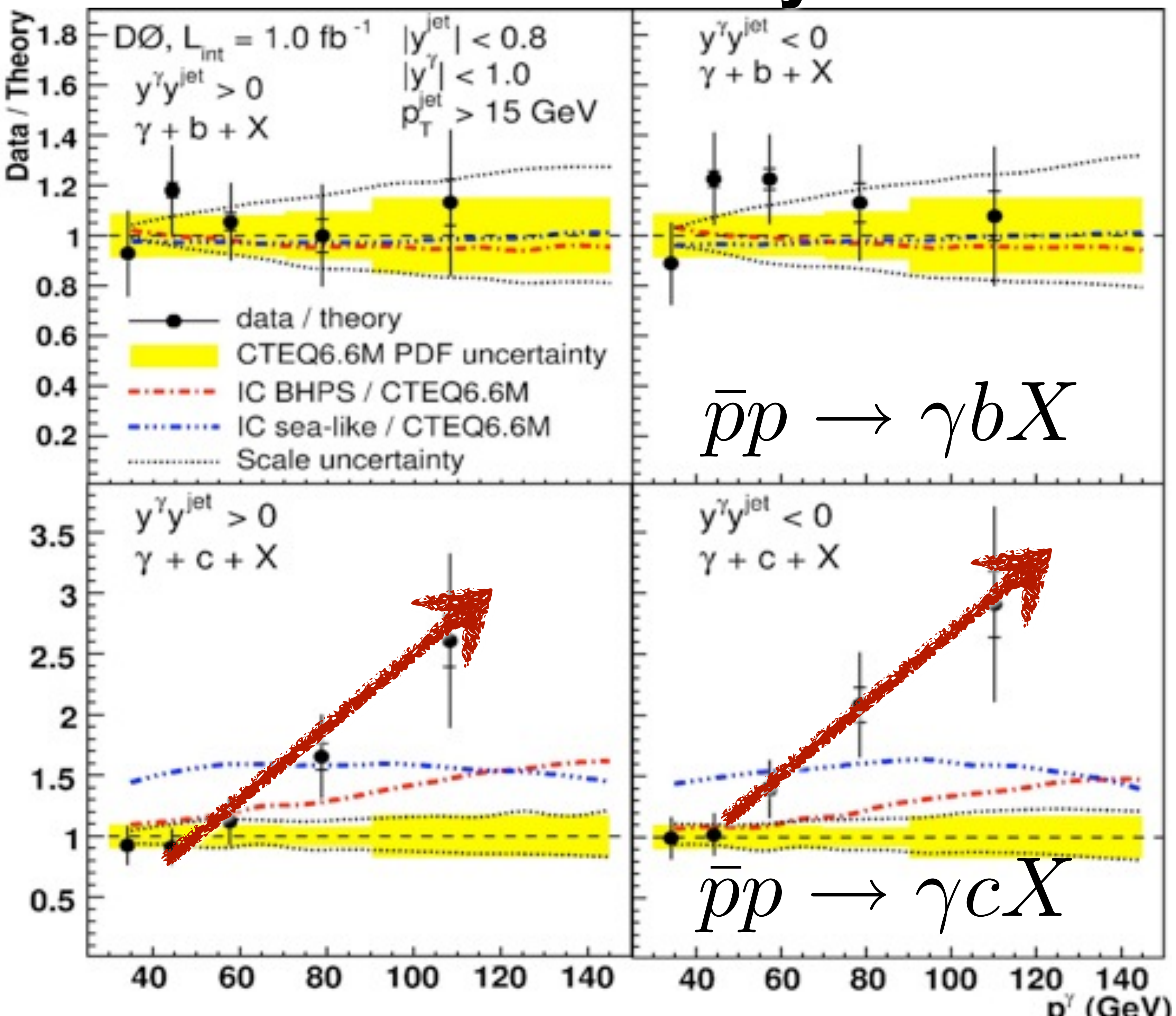
DGLAP / Photon-Gluon Fusion: factor of 30 too small

Two Components (separate evolution):

$$c(x, Q^2) = c(x, Q^2)_{\rm extrinsic} + c(x, Q^2)_{\rm intrinsic}$$

Measurement of $\gamma + b + X$ and $\gamma + c + X$ Production Cross Sections in $p\bar{p}$ Collisions at $\sqrt{s} = 1.96$ TeV

Data/Theory



$$\frac{\Delta\sigma(\bar{p}p \to \gamma c X)}{\Delta\sigma(\bar{p}p \to \gamma b X)}$$

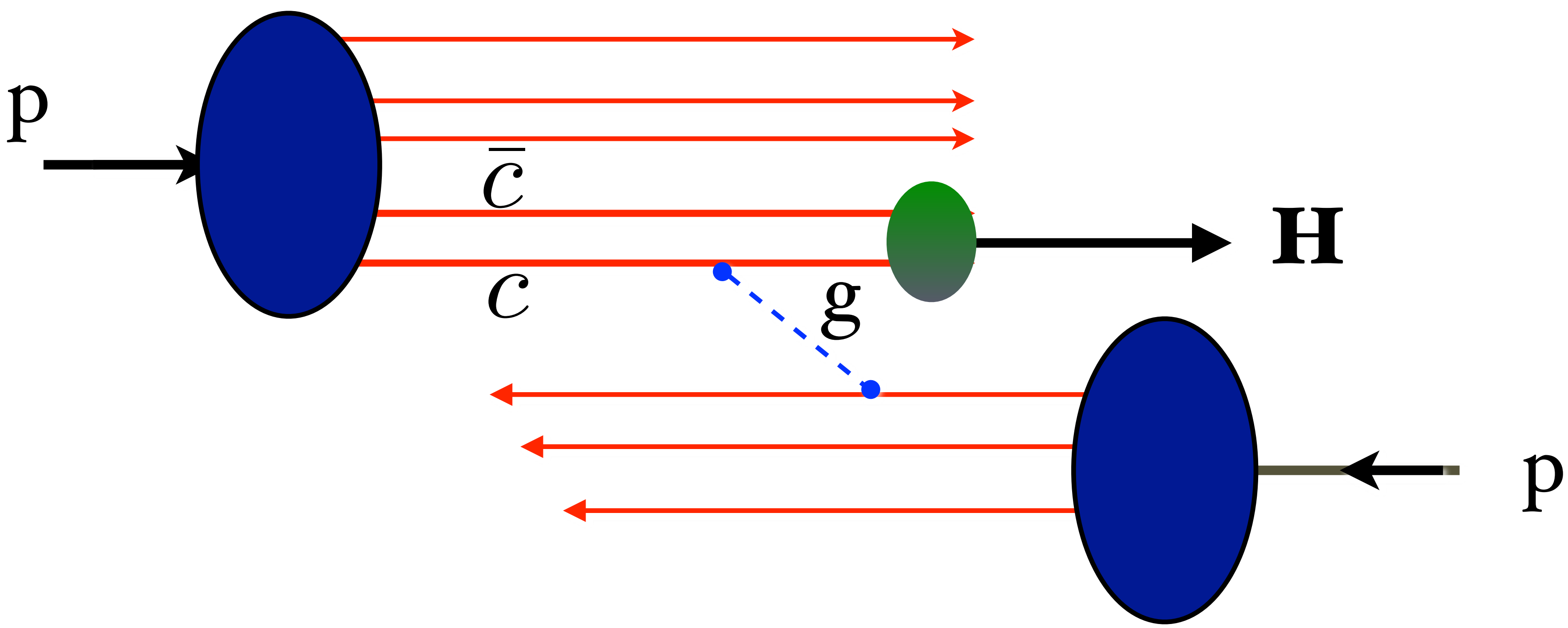
Ratio insensitive to gluon PDF, scales

Signal for significant IC at x > 0.1

Consistent with EMC measurement of charm structure function at high x

Goldhaber, Kopeliovich, Schmidt, Soffer sjb

Intrinsic Charm Mechanism for Inclusive High-X_F Higgs Production



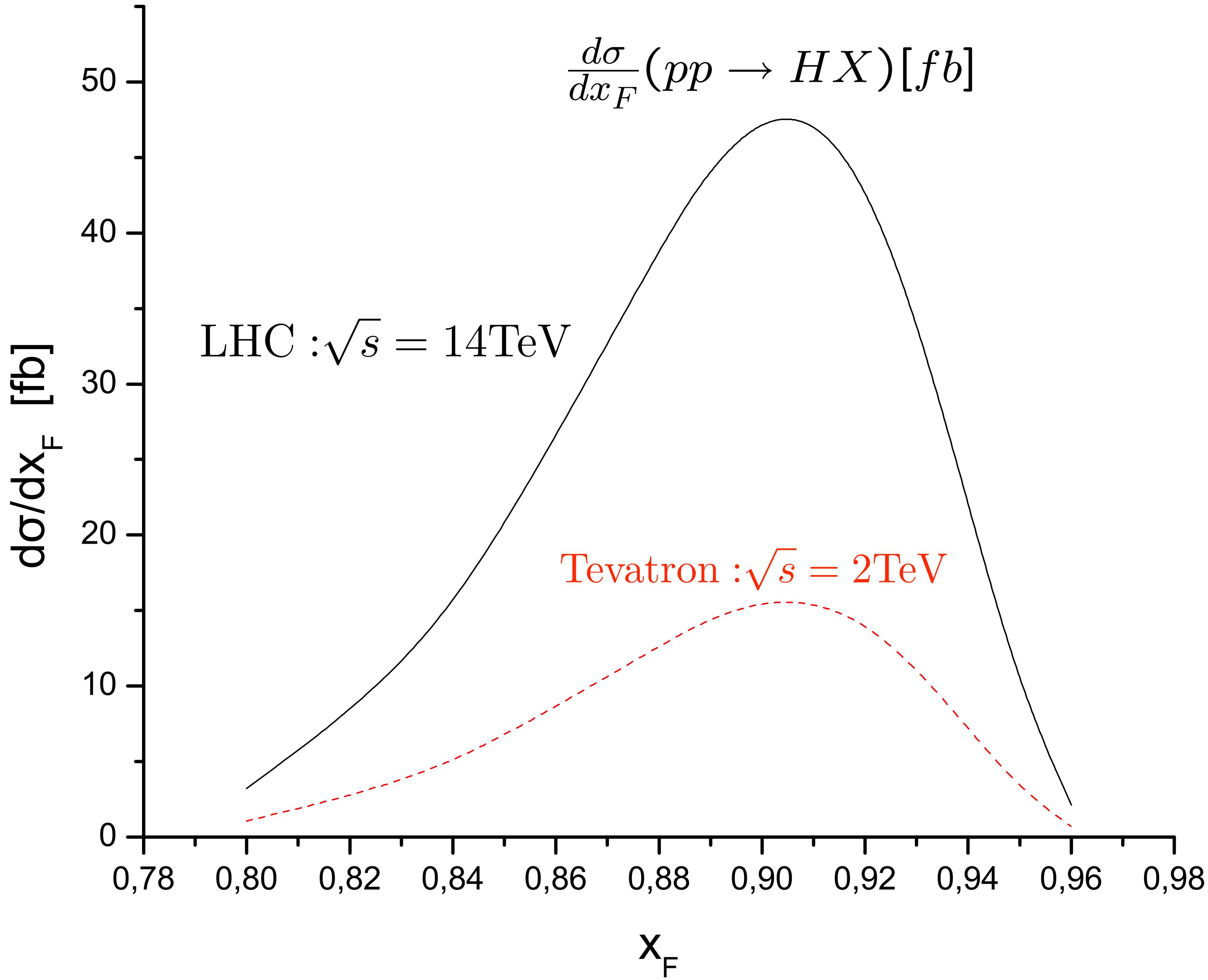
$$pp \to H X$$

Also: intrinsic strangeness, bottom, top

Higgs can have > 80% of Proton Momentum!

New production mechanism for Higgs

Intrinsic Heavy Quark Contribution to Inclusive Higgs Production



Goldhaber, Kopeliovich, Schmidt, sjb

Recent papers on PMC

Idea and initial application

- Brodsky and Wu, Phys.Rev.D85,034038(2012)
- Brodsky and Wu, Phys.Rev.D85,114040(2012)
- Brodsky and Wu, Phys.Rev.D86,014021(2012)
- Brodsky and Wu, Phys.Rev.D86,054018(2012)
- Brodsky and Giustino, Phys.Rev.D86,085026(2012)

- Brodsky and Wu, **Phys.Rev.Lett.**109,042002(2012)
- Matin, Brodsky and Wu, **Phys.Rev.Lett.**110,192001(2013)
- Wu, Brodsky and Matin, **Prog.Part.Nucl.Phys.**72,44(2013) (Invited Review)

Features and applications

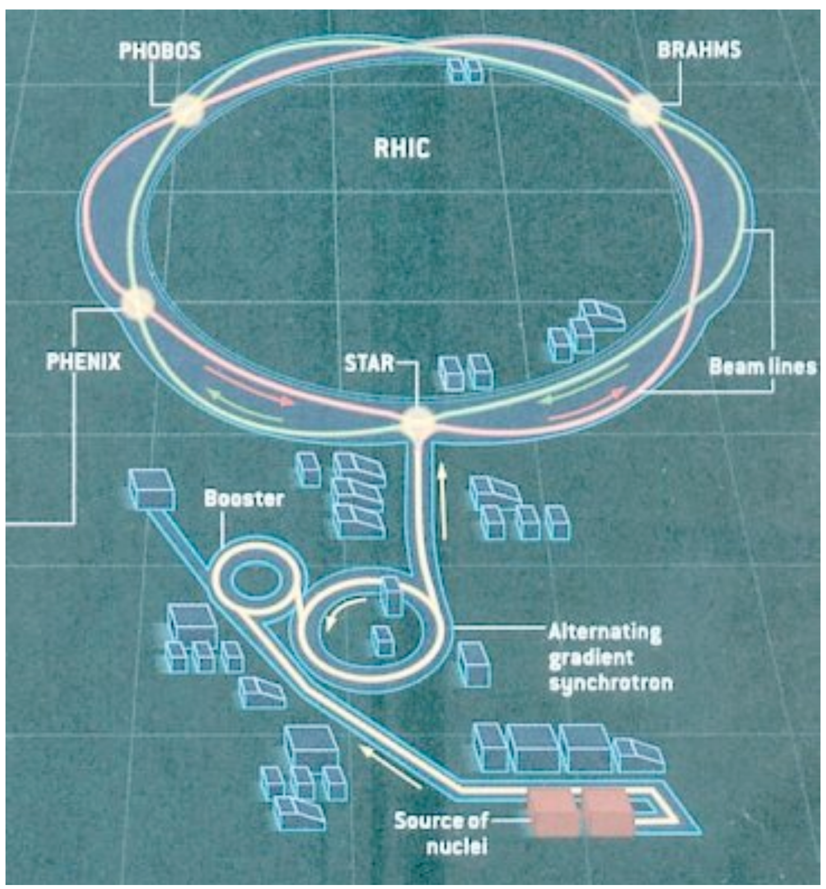
- Wang, Wu and etal., 1301.2992 (NPB876, 731(2013))
- Brodsky, Matin and Wu, 1304.4631 (PRD accepted(2014))
- Zheng, Wu and etal., 1308.2381 (JHEP10, 117(2013))
- Wang, Wu and etal., 1308.6364 (EPJC under review)
- Wang, Wu and etal., 1311.5108 (PRD under review)
- Chen, Wu and etal., 1311.2735 (PRD89,014006(2014))
- Sun, Wu and etal., 1401.2735(PRD under review)

Elimination of QCD Scale Ambiguities

The Principle of Maximum Conformality (PMC), and Novel QCD Effects

11th International Workshop on High P_T in the RHIC and LHC Era

April 12, 2016



Stan Brodsky

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DIE LUFT DER FREIHEIT WEHT
1891

with Leonardo Di Giustino , Xing-Gang Wu, and Matin Mojaza